Electronic Journal of Graph Theory and Applications **6** (1) (2018), 29–36

Electronic Journal of
Graph Theory and Applications

On classes of neighborhood resolving sets of a graph

B. Sooryanarayana, Suma A. S.

Department of Mathematical and Computational Studies
Dr. Ambedkar Institute of Technology, Bengaluru
Karnataka State, INDIA, Pin 560 056

dr_bsnrao@dr-ait.org, suma.ts6@gmail.com

Abstract

Let $G = (V, E)$ be a simple connected graph. A subset S of V is called a neighbourhood set of G if $G = \bigcup_{s \in S} \langle N[s] \rangle$, where $N[v]$ denotes the closed neighbourhood of the vertex v in G. Further for each ordered subset $S = \{s_1, s_2, \ldots, s_k\}$ of V and a vertex $u \in V$, we associate a vector $\Gamma(u/S) = (d(u, s_1), d(u, s_2), \ldots, d(u, s_k))$ with respect to S, where $d(u, v)$ denote the distance between u and v in G. A subset S is said to be resolving set of G if $\Gamma(u/S) \neq \Gamma(v/S)$ for all $u, v \in V - S$. A neighbouring set of G which is also a resolving set for G is called a neighbourhood resolving set (nr-set). The purpose of this paper is to introduce various types of nr-sets and compute minimum cardinality of each set, in possible cases, particularly for paths and cycles.

Keywords: resolving set, neighbourhood set, neighbourhood resolving sets.
Mathematics Subject Classification : 05C20
DOI: 10.5614/ejgta.2018.6.1.3

1. Introduction

All the graphs considered in this paper are connected, simple, undirected, and finite. Let p_1 be a graph property satisfied by at least one subset of vertices of G. Then such subsets S which satisfies the property p_1 are called p_1-sets of G. A p_1-set S of G is called a P_1-set if $\bar{S}$ is not a p_1-set of G. A $p_1^\star$-set of G is a set S such that both S and $\bar{S}$ are p_1-sets of G. A $P_1^\star$-set of G is a

Received: 5 June 2016, Revised: 24 December 2017, Accepted: 3 January 2018.

set S such that both S and $\bar{S}$ are not p_1-sets of G. If p_2 is another graph property satisfied by any subset of vertices of G, then a set S which satisfies both the property p_1 and p_2 is called a p_1p_2-set. If S is a p_1-set and also a $p_2^\star$-set, then we say S is a $p_1p_2^\star$-set. Similarly, $p_1p_2p_3$-sets, $p_1P_2^\star p_3$-sets, $p_1P_2P_3^\star$-sets, etc., are defined.

A pq-set is said to be a minimal pq-set of G if none of its proper subsets are pq-set of G. The minimum cardinality of a minimal pq-set of G is called lower pq number of G and is denoted by $l_{pq}(G)$.

Let G be a graph and v be a vertex of G. Let $N(v)$ be the set of vertices adjacent to v in G and $N[v] = N(v) \cup \{v\}$. A subset S of vertex set of G is called a neighbourhood set or an n-set of G if $G = \bigcup_{v \in S} \langle N[v] \rangle$, where $\langle N(v) \rangle$ is the subgraph of G induced by the set S. Further a subset S of a vertex set of G is called a resolving set or an r-set of G if for each pair $u, v \notin S$ there is a vertex $w \in S$ with the property that $d(v, w) \neq d(u, w)$.

The metric dimension of G, denoted by $\beta(G)$, is the minimum cardinality of all the resolving sets of G. A resolving set with minimum cardinality is called a *metric basis*. The concept of Metric dimension was introduced by F. Harary and R.A. Melter [3] and independently by P.J. Slater [13] under the term locating set. For more works on metric dimension, we refer [2, 5, 6, 7, 10, 11, 12, 14, 15].

The neighbourhood number of a graph was introduced by E. Sampathkumar et al. in [8] and studied the relationship of $l_n(G)$ (denoted by n_0) with some other known graph parameters.

If S is both neighbourhood and resolving, then in the above notation we write S as an nr-set. The terms not defined here may found in [1]. Throughout this paper P_k denotes a path on k vertices with a vertex set $V = \{v_i : 1 \leq i \leq k\}$ and an edge set $E = \{v_iv_{i+1} : 1 \leq i \leq k-1\}$. Similarly, C_k denotes a cycle on k vertices with a vertex set $V = \{v_i : 1 \leq i \leq k\}$ and an edge set $E = \{v_iv_{i+1}\} \bigcup \{v_1v_k\}$.

Remark 1.1. From the definition of a resolving set, it is clear that any 2-element subset of vertices of a path P_k is always an r-set of P_k. In fact, if $S = \{a, b\}$ and u, v be arbitrary vertices of P_k such that $d(u, a) = d(v, a)$, then a is the central vertex of the uv-path in P_k, but then exactly one of the paths, ub-path or vb-path, in P_k contains the vertex a and hence $d(u, b) \neq d(v, b)$.

Remark 1.2. A singleton set $S = \{v\}$ is a resolving set of a path P if and only if v is an end vertex of P_k.

Remark 1.3. A subset of vertices of P_k containing an end vertex is always a resolving set of P_k.

Remark 1.4. For a connected graph G of order k, every subset of cardinality at least $k-1$ is always an n-set.

Remark 1.5. Since a superset of any r-set of a graph G is also an r-set of the graph G, it follows from Remark 1.1 that every i-element subset of the vertex set of a path P_k is always an r-set of P_k, for every i, $2 \leq i \leq k$.

Observation 1.1. *Every n-set of a path P_k has at least 2 elements, whenever $k \geq 4$.*

Observation 1.2. *Every r-set of a path P_k, $2 \leq k \leq 3$, contains a pendent vertex.*

We recall the following for immediate reference;

Theorem 1.1 (S. Khuller, B. Raghavachari, and A. Rosenfeld [6]). *For a simple connected graph G, $\beta(G) = 1$ if and only if $G \cong P_k$.*

Theorem 1.2 (F. Harary and R.A.Melter [3]). *For any integer $k \geq 3$, the metric dimension of a cycle on k vertices is 2.*

Theorem 1.3 (B. Sooryanarayana [14]). *A graph G with $\beta(G) = k$, cannot contain $k_{2^k+1} - (2^{k-1} - 1)e$ as a subgraph.*

Theorem 1.4 (E. Sampathkumar and Prabha S. Neeralagi [9]). *For a path P_k on k vertices, the lower neighbourhood number $l_n(P_k) = \lfloor \frac{k}{2} \rfloor$.*

Theorem 1.5 (E. Sampathkumar and Prabha S. Neeralagi [8]). *For a cycle C_k of length $k \geq 4$, the lower neighbourhood number $l_n(C_k) = \lceil \frac{k}{2} \rceil$.*

Theorem 1.6 (E. Sampathkumar and Prabha S. Neeralagi [8]). *A set S of vertices of a graph G is an n-set if and only if every line of $\langle V(G) - S \rangle$ belongs to a triangle one of whose vertices belong to S.*

2. nr-sets and Dimensions of a Path

Theorem 2.1. *For any integer $k \geq 1$, $l_{nr}(P_k) = \begin{cases} \lceil \frac{k}{2} \rceil, & \text{for} \quad k \leq 3, \\ \lfloor \frac{k}{2} \rfloor, & \text{for} \quad k \geq 4. \end{cases}$*

Proof. For the case $k = 1, 2$, it is easy to see that any singleton subset of $V(P_k)$ is always an nr-set. For $k = 3$, a singleton subset containing an end vertex is not an n-set and a singleton subset containing the central vertex is not an r-set of P_3. Therefore, every nr-set should have at least two elements. Further, as any subset $S \subseteq V(P_3)$ with $|S| = 2$ is an nr-set for $P_3, l_{nr}(P_3) = 2$. Now for $k \geq 4$, any subset $S \subseteq V(P_k)$ containing two or more elements is always an r-set (by Remark 1.5). Therefore, as $l_n(P_k) \geq 2$ for all $k \geq 4$, it follows that $l_{nr}(P_k) = l_n(P_k) = \lfloor \frac{k}{2} \rfloor$(by Theorem 1.4). □

Theorem 2.2. *For any integer $k \geq 1$, $l_{nR}(P_k) = \begin{cases} k, & \text{for} \quad k = 1, 2, \\ k-1, & \text{for} \quad k \geq 3. \end{cases}$*

Proof. Let S be an nR-set of a path P_k. Then S is an r-set and $\bar{S}$ is not an r-set. So, by Remark 1.1 and Remark 1.3, it follows that a minimal R-set S should contain both the end vertices and is of cardinality at least $k - 1$ whenever $k \geq 3$ or exactly k if $k \leq 2$. But then, by Remark 1.4, S is an n-set of P_k. Hence $l_{nR} = k - 1$ if $k \geq 3$ or $l_{nR} = k$ if $k \leq 2$. □

Theorem 2.3. *For any integer $k \geq 1$, $l_{NR}(P_k) = \begin{cases} k, & \text{for} \quad k \leq 2, \\ k-1, & \text{for} \quad k \geq 3. \end{cases}$*

Proof. Follows by the proof of the previous Theorem 2.2, as each nR-set S of P_k is also an NR-set of P_k (Since the set $\bar{S}$ contains at most one element which is non-end vertex and hence by Observation 1.1 and Observation 1.2, $\bar{S}$ is not an n-set if $k \neq 3$ and not an r-set if $k = 3$). □

Lemma 2.1. *Any independent set S of vertices of a path P_k contains more than $\frac{k}{2}$ vertices is always an n-set.*

Proof. Let S be an independent set of the path P_k contains more than $\frac{k}{2}$ vertices. Then k is odd, $S = \{v_1, v_3, v_5, \ldots, v_{k-2}, v_k\}$, and $\bigcup_{v \in S} N[v] = V(P_k)$. Let $e_i = v_i v_{i+1}$ be an edge of P_k, $1 \leq i \leq k-1$. Then e_i is an edge of either $\langle N[v_i] \rangle$ or $\langle N[v_{i+1}] \rangle$ depending upon whether i is odd or even. Hence for each i, the edge $e_i \in \langle N[v_j] \rangle$ for some odd j. Therefore, $\bigcup_{v_i \in S} \langle N[v_i] \rangle = G$. □

Similarly, we prove:

Lemma 2.2. *Any independent set S of vertices of a path P_{2k} contain (at least) k vertices is always an n-set of P_{2k}.*

Lemma 2.3. *If S is an n-set of the graph G, then $\bar{S}$ is independent.*

Proof. If not, suppose that $\bar{S}$ contains two adjacent vertices say x and y, then the edge xy is not in the graph $\bigcup_{v \in S} \langle N[v] \rangle = G$, a contradiction to the fact that S is an n-set. □

Theorem 2.4. *For any integer,* $l_{Nr}(P_k) = \begin{cases} k, & \text{for} \quad k = 1, 2, \\ \lceil \frac{k}{2} \rceil, & \text{for} \quad k \geq 3. \end{cases}$

Proof. The result is obvious for $k \leq 4$. Consider the case $k \geq 5$, let S be an N-set of P_k. Then S is an n-set, so by Theorem 1.4, $|S| \geq \lfloor \frac{k}{2} \rfloor \geq 2$ vertices and hence by Remark 1.5, S is also an r-set. If k is odd and $|S| = \lfloor \frac{k}{2} \rfloor$, then $|\bar{S}| \geq \lfloor \frac{k}{2} \rfloor$, so by Lemma 2.3 and Lemma 2.1 the subset $\bar{S}$ is an n-set, a contradiction to the fact that S is an N-set. Therefore, $|S| \geq \lceil \frac{k}{2} \rceil$ for all k implies that $l_{Nr}(P_k) \geq \lceil \frac{k}{2} \rceil$. On the other hand, it is easy to see that the set $S = \{v_{2\lfloor \frac{k}{4} \rfloor}, v_{2\lfloor \frac{k}{4} \rfloor - 2}, \ldots, v_2\} \bigcup \{v_P\} \bigcup \{v_{\lfloor \frac{k}{2} \rfloor + 1}, v_{\lfloor \frac{k}{2} \rfloor + 3}, \ldots, v_{k-1}\}$ is an Nr-set of P_k with $|S|$=$\lceil \frac{k}{2} \rceil$ where $p = 2$, if k is even and $p = 1$, if k is odd. Thus, $l_{Nr}(P_k) \leq \lceil \frac{k}{2} \rceil$. □

Theorem 2.5. *For any positive integer k, $k \neq 1, 3$, $l_{n^\star r}(P_k) = l_{nr^\star}(P_k) = l_{n^\star r^\star}(P_k) = \lfloor \frac{k}{2} \rfloor$.*

Proof. The result is obvious for $k = 2$. Now for the case $k \geq 4$, as every $n^\star$-set S is also an n-set, we have $|S| \geq \lfloor \frac{k}{2} \rfloor$ (by Theorem 1.4) and hence $l_{n^\star r^\star}(P_k),\ l_{n^\star r}(P_k),\ l_{nr^\star}(P_k) \geq \lfloor \frac{k}{2} \rfloor$. On the other hand, we see that the set $S = \{v_2, v_4, \ldots, v_{2\lfloor \frac{k}{2} \rfloor}\}$ is an n-set of P_k. So, by Lemma 2.1 or Lemma 2.2 respectively when k is odd or even, the set $\bar{S}$ is an n-set. Since $k \geq 4$, both S and $\bar{S}$ have at least two elements and hence each of them will resolve P_k. Hence S is an $n^\star r$-set as well as $nr^\star$-set and $n^\star r^\star$-set with $|S| = \lfloor \frac{k}{2} \rfloor$. Therefore, $l_{n^\star r}(P_k) \leq \lfloor \frac{k}{2} \rfloor, l_{nr^\star}(P_k) \leq \lfloor \frac{k}{2} \rfloor$, and $l_{n^\star r^\star}(P_k) \leq \lfloor \frac{k}{2} \rfloor$. □

Remark 2.1. When $k = 1$, $\bar{S}$ is empty. Hence $n^\star$-set as well as $r^\star$-set are not defined. But when $k = 3$, it is easy to see that $l_{n^\star r}(P_3) = l_{nr^\star}(P_3) = 2$. However, P_3 has no $n^\star r^\star$-set S and hence $l_{n^\star r^\star}(P_3)$ is not defined.

Theorem 2.6. *For any integer $k \geq 4$, $l_{N^\star r}(P_k) = l_{N^\star r^\star}(P_k) = 2$.*

Proof. Let S be an $N^\star r$-set of P_k. Then S is not an n-set, $\bar{S}$ is not an n-sets, and S is an r-set. Now, if $|S| = 1$, then S contains only an end vertex of P_k (by Remark 1.2) and hence $|\bar{S}| = k - 1$. But then, $\bar{S}$ is an n-set (by Remark 1.4), a contradiction. Thus, $2 \leq |S| \leq k - 2$. Hence $l_{N^\star r}(P_k) \geq 2$ and $l_{N^\star r^\star}(P_k) \geq 2$. On the other hand, take $S' = \{v_1, v_2\}$. The set S' as well as $\bar{S}'$ are not n-sets (since the edge v_1v_2 is not an edge of $\bigcup_{v \in \bar{S}'} \langle N[v] \rangle$). But S' is an r-set (and $\bar{S}'$ is also an r-set), whenever $k \geq 4$ (since $|S'| = 2$ and $|\bar{S}'| \geq 2$ and by Remark 1.5). Hence $l_{N^\star r}(P_k) \leq 2$ and $l_{N^\star r^\star}(P_k) \leq 2$. □

Remark 2.2. If $k \leq 3$, for every subset S of $V(P_k)$, either S or $\overline{S}$ is an n-set. Hence no $N^\star$-set exists.

We end up this section with the following theorem, whose proof follows similar to the proof of Theorem 2.4.

Theorem 2.7. *For any integer $k \geq 3$, $l_{Nr^\star}(P_k) = \lceil \frac{k}{2} \rceil$.*

When $k = 1$, no $r^\star$-set exists and when $k = 2$, no N-set exists. It is easy to see that the other sets like $nR^\star$-set, $n^\star R^\star$-set, $NR^\star$-set, and $N^\star R^\star$-set are not exists in any path due to the non-existence of $R^\star$-sets. Finally, the non-existence of $N^\star R$-set is due to the fact that if S is any such set, then its complement should contains exactly one vertex other than the end vertex to become an R-set implies that the set S is an n-set (so not an $N^\star$-set).

3. nr-sets and Dimensions of a Cycle

We first restate the consequences of Theorem 1.6 as;

Lemma 3.1. *Let $e = xy$ be an edge of a graph G such that e is not an edge of a triangle in G and S be an n-set of G. Then $x, y \in N[v]$ for some $v \in S$ if and only if $x = v$ or $y = v$.*

Lemma 3.2. *If S is an n-set of a graph G, then for each edge $e = xy$ there exists a vertex v in S such that both $x, y \in N[v]$.*

Theorem 3.1. *For each integer $i \geq 3$, every i-element subset S of vertices of a cycle C_k is always an r-set.*

Proof. Let S be a subset of the vertices of C_k with cardinality at least 3. Let $a, b, c \in S$ and x, y be any two vertices of cycle C_k for $k \geq 3$. If possible, let $d(a, x) = d(a, y)$ and $d(b, x) = d(b, y)$. Then a and b lie in distinct xy-paths in C_k and C_k is an even cycle. In case if c lies between a and x, then $d(c, x) < d(c, y)$ and hence c resolves the pair x, y. Similarly, other cases follows by symmetry. □

Remark 3.1. A set containing two adjacent vertices of a cycle C_k is always an r-set of C_k for each $k \geq 3$.

Theorem 3.2. *For any integer $k \geq 3$,* $l_{nr}(C_k) = \begin{cases} 3, & \textit{for } k = 4, \\ \lceil \frac{k}{2} \rceil, & otherwise. \end{cases}$

Proof. In the case $k = 4$, it follows by Theorem 1.4 that $|S| \geq 2$. If $|S| = 2$, then S contains two adjacent vertices (else it is not an r-set). But then, $\langle V(C_4) - S\rangle$ contains an edge and hence by Theorem 1.6, C_k should contain a triangle, a contradiction. Hence every nr-set should have at least 3 elements. For the case $k \geq 5$, it is easy to see from Theorem 1.5 and Theorem 1.6 that the set $S = \{v_1, v_3, v_5, \ldots, v_{2\lceil \frac{k}{2}\rceil -1}\}$ is an n-set and hence by Theorem 3.1, it follows that $l_{nr}(C_k) = |S| = \lceil \frac{k}{2}\rceil$. □

Theorem 3.3. *For any integer $k \geq 4$, $l_{N^\star r}(C_k) = l_{N^\star r^\star}(C_k) = 2$*

Proof. Let $e = xy$ be an edge of C_k and $S = \{x, y\}$. Then S is a resolving set for C_k. Now as $k \geq 4$, there is an edge $e_1 = uv$ not adjacent to e. So, by Lemma 3.2, S is not an n-set (Since C_k has no triangle and $u, v \notin S$). Hence S is an $N^\star r$-set. Further as $\beta(C_k) = 2$, there are no singleton r-sets implies that the above set S is a minimal $N^\star r$-set, $l_{N^\star r}(C_k) = 2$. Also, $\bar{S}$ contains at least 3 vertices if $k > 4$ and 2 adjacent vertices if $k = 4$. So, by Theorem 3.1 and Remark 3.1, $\bar{S}$ is an r-set. Therefore, S is also an $N^\star r^\star$-set of minimum cardinality, so $l_{N^\star r^\star}(C_k) = 2$ for all $k \geq 4$. □

Lemma 3.3. *Let S be a minimal n-set of a graph G with $\Delta(G)$=2 and H=$\langle S\rangle$. Then $\Delta(H) < 2$.*

Proof. If possible, let S be a minimal n-set of G and $\Delta(H) = 2$. Then there exists $a, b, c \in S$, Such that $ab, bc \in E(G)$. Consider the set $S' = S - \{b\}$. Since $\Delta(G) = 2$, we have $deg_G(b) = 2$ and hence b is adjacent to only a and c. Therefore, S' covers all the edges of G incident with b as well as other edges of G (Since other edges covered by S). This shows that S' is an n-set, a contradiction to the minimality of S. □

Theorem 3.4. *For any integer $k > 4, l_{Nr}(C_k) = l_{Nr^\star}(C_k) = \lceil \frac{k+1}{2}\rceil$. Also, $l_{Nr}(C_4) = 3$.*

Proof. Let S be a minimal Nr-set of cycle C_k, $k > 4$. Then S is an n-set, therefore by Theorem 1.5, $|S| \geq \lceil \frac{k}{2}\rceil$ and by Lemma 3.3 the induced subgraph $\langle S\rangle$ has no two adjacent edges of G (i.e $deg_{\langle S\rangle}(v) \leq 1, \forall v \in S$). So, if k is even and $|S| = \lceil \frac{k}{2}\rceil$, then in the view of Lemma 3.2, we have, $\bar{S}$ is an n-set, a contradiction to the fact that S is an N-set. Thus, $|S| \geq \lceil \frac{k+1}{2}\rceil$ implies that $l_{Nr}(C_k) \geq \lceil \frac{k+1}{2}\rceil$ and $l_{Nr^\star}(C_k) \geq \lceil \frac{k+1}{2}\rceil$. On the other hand, consider the set $S = \{v_1, v_3, v_5, \ldots, v_{2\lceil \frac{k+1}{2}\rceil -3}\} \bigcup \{v_{k-1}\}$. The set S is an n-set with $|S| = \lceil \frac{k+1}{2}\rceil$ and $|\bar{S}| = \lfloor \frac{k-1}{2}\rfloor < \lceil \frac{k}{2}\rceil$ and hence $\bar{S}$ is not an n-set implies that S is an N-set. Finally, as $k > 4$, we have $|S| > 3$. Hence by Theorem 3.1, S is also an r-set. Thus, $l_{Nr}(C_k) \leq \lceil \frac{k+1}{2}\rceil$. Further when $k = 5$, it is easy to see that $\bar{S}$ contains an adjacent pair of vertices and when $k > 5$, the set $\bar{S}$ has at least 3 vertices. Hence by Remark 3.1 and the 3.1, the set S is also an $r^\star$-set. Hence it also follows that $l_{Nr^\star}(C_k) \leq \lceil \frac{k+1}{2}\rceil$. Lastly, the case $k = 4$ follows easily. □

Remark 3.2. When $k = 3$, it is easy to see that for every nr-set S of C_3, the set $\bar{S}$ is also an n-set and no N-set exists.

Theorem 3.5. *For any integer $k > 4$, $l_{nr^\star}(C_k) = \lceil \frac{k}{2}\rceil$*

Proof. Follows immediately by Theorem 1.4 and Theorem 3.1, as $l_{nr^\star}(C_k) = l_n(C_k) = \lceil \frac{k}{2}\rceil$ for all $k > 4$. □

Remark 3.3. Since $\beta(C_k) = 2$, every r-set of C_k should have at least 2 elements. Therefore, for the existence of an $r^\star$ set of a cycle C_k, k should be at least 5. Further when $k = 3$ or 4, it is easy to see that for every nr-set S of C_k we get $|\bar{S}| = 1$, and hence S is not an $r^\star$-set.

Theorem 3.6. *For any integer* $k \geq 4$, $l_{NR}(C_k) = l_{nR}(C_k) = \begin{cases} k-2, & \text{when } k \text{ is even and } k \neq 4, \\ k-1, & \text{otherwise.} \end{cases}$

Proof. Since $\beta(C_k) = 2$, any two vertices of C_k resolves C_k except the case k is even and the vertices are diagonally opposite. Therefore, for $k > 4$, every R-set S should have minimum of $k-1$ vertices whenever k is odd and $k-2$ if k is even. In either of the cases, the subgraph $\bigcup_{v \in S} N[v] \cong C_k$ for every R-set S and $\bigcup_{v \in \overline{S}} N[v] \neq C_k$ for $k \neq 4$ and hence S is an n-set as well as an N-set. When k=4, every N-set should have at least 3 elements and such a set S with $|S| = 3$ is always an R-set. □

Theorem 3.7. *For every integer* $k \geq 3$, $l_{n^\star r^\star}(C_{2k}) = l_{n^\star r}(C_{2k}) = k$.

Proof. Let S be an $n^\star$-set. Then S and $\overline{S}$ both are edge covering of C_{2k}. Since edge covering number of C_{2k} is $k, |S| = |\bar{S}| = k$. Also, both S and $\bar{S}$ are r-sets (since $k \geq 3$). Finally, every maximal independent set S is an $n^\star r^\star$-set as well as $n^\star r$-set. Hence the result. □

Remark 3.4. For an odd cycle, no $n^\star$-set exists as each n-set contains both end vertices of an edge (so $\bar{S}$ is not an n-set, by Lemma 3.2).

Acknowledgement

The authors are very much thankful to the Management, and the Principal Dr. Ambedkar Institute of Technology, Bengaluru, for their constant support during the preparation of this paper. Also special thanks to the anonymous referees for their suggestions for the improvement of this paper.

References

[1] F. Buckley and F. Harary, *Distance in graphs*, Addison-Wesley, 1990.

[2] G. Chartrand, L. Eroh, Mark A. Johnson and O. Oellermann, Resolvability in graphs and the metric dimension of a graph, *Discrete Appl. Math.* **105** (2000), 99–113.

[3] F. Harary and R.A. Melter, On the metric dimension of a graph, *Ars Combinatoria* **2** (1976), 191–195.

[4] N. Hartsfield and G. Ringel, *Pearls in graph theory*, Academic Press, USA, 1994.

[5] M. Imran, Abdul Q. Baig, S.A. Bokhary and E.T. Baskoro, New classes of convex polytopes with constant metric dimension, *Utilitas Mathematica* **95** (2014), 97–111.

[6] S. Khuller, B. Raghavachari and A. Rosenfeld, Land marks in graphs, *Disc. Appl. Math.* **70** (1996), 217–229.

[7] V. Saenpholphat and P. Zhang, Connected resolvability of graphs, *Australas. J. Comb.* **28** (2003), 25–37.

[8] E. Sampathkumar and Prabha S. Neeralagi, The neighbourhood number of a graph, *Indian J. Pure. Appl. Math.*, **16** (2) (1985), 126–132.

[9] E. Sampathkumar and Prabha S. Neeralagi, The independent, perfect and connected neighbourhood numbers of a graph, *J. Combin. Inform. System Sci.* **19** (1994), 139–145.

[10] S.W. Saputro, R. Simanjuntak, S. Uttunggadewa, H. Assiyatun, E.T. Baskoro, A.N.M. Salman and M. Baca, The metric dimension of the lexicographic product of graphs, *Discrete Math.* **313** (9) (2013), 1045–1051.

[11] A. Seb and E. Tannier, On metric generators of graphs, *Math. Opr. Res.* **29** (2) (2004), 383–393.

[12] B. Shanmukha, B. Sooryanarayana and K.S. Harinath, Metric dimension of wheels, *Far East J. Appl. Math.* **8** (3) (2002), 217–229.

[13] P.J. Slater, Leaves of trees, *Congres. Numer.* **14** (1975), 549–559.

[14] B. Sooryanarayana, On the metric dimension of graph, *Indian. J. Pure Appl. Math.* **29** (4) (1998), 413–415.

[15] B. Sooryanarayana and B. Shanmukha, A note on metric dimension, *Far East J. Appl. Math.* **5** (3) (2001), 331–339.