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The eccentric-distance sum of some graphs

Padmapriya P., Veena Mathad

Department of Studies in Mathematics
University of Mysore, Manasagangotri
Mysuru - 570 006, India

padmapriyap7@gmail.com, veena_mathad@rediffmail.com

Abstract

Let $G = (V, E)$ be a simple connected graph. The eccentric-distance sum of G is defined as $\xi^{ds}(G) = \sum_{\{u,v\}\subseteq V(G)} [e(u) + e(v)]d(u, v)$, where $e(u)$ is the eccentricity of the vertex u in G and $d(u, v)$ is the distance between u and v. In this paper, we establish formulae to calculate the eccentric-distance sum for some graphs, namely wheel, star, broom, lollipop, double star, friendship, multi-star graph and the join of P_{n-2} and P_2.

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1. Introduction

Let G be a simple connected graph with the vertex set $V(G)$ and the edge set $E(G)$. The degree of a vertex $u \in V(G)$ is denoted by $d(u)$ and is the number of vertices adjacent to u. For vertices $u, v \in V(G)$, the distance $d(u, v)$ is defined as the length of any shortest path connecting u and v in G and $D(u)$ denotes the sum of distances between u and all other vertices of G. The eccentricity $e(u)$ of a vertex u is the largest distance between u and any other vertex v of G, i.e., $e(u) = max\{d(u, v); v \in V(G)\}$. Let K_n, P_n, W_n, C_n and $K_{1,n-1}$ denote a complete graph, path, wheel, cycle and star on n vertices, respectively [14].

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The *Wiener index* is defined as the sum of all distances between unordered pairs of vertices

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d(u, v).$$

It is considered as one of the most used topological index with high correlation with many physical and chemical indices of molecular compounds (for the recent survey on Wiener index see [4, 5]).

The parameter $DD(G)$ called the degree distance of G was introduced by Dobrynin and Kochetova [6] and Gutman [13] as a graph-theoretical descriptor for characterizing alkanes; it can be considered as a weighted version of the Wiener index and is defined as

$$DD(G) = \sum_{\{u,v\} \subseteq V(G)} [d(u) + d(v)]d(u, v) = \sum_{u \in V(G)} d(u)D(u)$$

where the summation goes over all pairs of vertices in G. In fact, when G is a tree on n vertices, it has been demonstrated that Wiener index and degree distance are closely related by (see [15, 19]) $DD(G) = 4W(G) - n(n-1)$.

Sharma, Goswami and Madan [26] introduced a distance-based molecular structure descriptor, *eccentric connectivity index (ECI)* defined as

$$\xi^c(G) = \sum_{v \in V(G)} e(v)d(v).$$

The index $\xi^c(G)$ was successfully used for mathematical models of biological activities of diverse nature [9, 11, 12, 22, 25, 26]. The investigation of its mathematical properties started only recently (for a survey on eccentric connectivity index see [17]). In [8, 18, 23, 28], the extremal graphs in various class of graphs with maximal or minimal ECI are determined. In [1, 2, 7] the authors determined the closed formulae for the eccentric connectivity index of nanotubes and nanotori.

Recently, a novel graph invariant for predicting biological and physical properties eccentric-distance sum was introduced by S.Gupta, M.Singh and A.K.Madan [12]. This topological index has vast potential application in structure activity/property relationships of molecules and it also displays high discriminating power with respect to both biological activities and physical properties; see[12]. The authors in [12] have shown that some structure activity and quantitative structure property studies using eccentric-distance sum were better than the corresponding values obtained using the Wiener index. It is also interesting to study the mathematical property of this topological index. The *eccentric-distance sum (EDS)* of G is defined as

$$\xi^{ds}(G) = \sum_{u \in V(G)} e(u)D(u).$$

The eccentric-distance sum can be defined alternatively as

$$\xi^{ds}(G) = \sum_{\{u,v\} \subseteq V(G)} [e(u) + e(v)]d(u, v).$$

Yu, Feng and Ilić [27] identified the extremal unicyclic graphs of given girth having the minimal and second minimal EDS; they also characterized trees with minimal EDS among the n-vertex trees of a given diameter. Hua, Xu and Shu[16] obtained the sharp lower bound on EDS of n-vertex cacti. Ilić, Yu and Feng [20] studied various lower and upper bounds for the EDS in terms of other graph invariant including the Wiener index, the degree distance index, the eccentric connectivity index and so on.

In this paper we establish formulae to calculate the eccentric-distance sum for some graphs, namely wheel, star, broom graph, lollipop, double star, friendship, multi-star graph and Pl_n graph.

2. Main Results

M. J. Morgan et al. calculated the eccentric-distance sum for complete graph, cycle graph and path graph.

Proposition 2.1. *[3]*
1.$\xi^{ds}(K_n) = n(n-1)$

$$2.\xi^{ds}(C_n) = \begin{cases} \dfrac{n^4}{8}, & \textit{if } n \textit{ is even} \\ \dfrac{n(n-1)^2(n+1)}{8}, & \textit{if } n \textit{ is odd.} \end{cases}$$

$$3.\xi^{ds}(P_n) = \begin{cases} \dfrac{25n^4 - 16n^3 - 28n^2 + 16n}{96}, & \textit{if } n \textit{ is even} \\ \dfrac{25n^4 - 16n^3 - 34n^2 + 16n + 9}{96}, & \textit{if } n \textit{ is odd.} \end{cases}$$

Proposition 2.2. $\xi^{ds}(W_n) = (n-1)(4n-9)$

Proof. $e(v_1) = 1$ and $e(v_i) = 2$ where, $i = 2, 3, \ldots, n$. Then,

$$\begin{aligned} \xi^{ds}(W_n) = & \sum_{\{v_i,v_j\}\subseteq V(W_n)} [e(v_i) + e(v_j)]d(v_i, v_j) \\ =&(1+2)1(n-1) + (2+2)1(2) + (2+2)2(n-4) \\ &+ (2+2)1 + (2+2)2(n-4) + (2+2)1 + (2+2)2(n-5) \\ &+ \cdots + (2+2)1 + (2+2)2(1) + (2+2)1 \\ =&(n-1)(4n-9). \end{aligned}$$

□

Proposition 2.3. $\xi^{ds}(K_{1,n-1}) = (n-1)(4n-5)$

Proof. $e(v_0) = 1$ and $e(v_i) = 2$ where, $i = 1, 2, 3, \ldots, n-1$. Then,

$$\begin{aligned}\xi^{ds}(K_{1,n-1}) =&(1+2)1(n-1) + (2+2)2(n-2) + (2+2)2(n-3) + \cdots + (2+2)2(1)\\ =&(n-1)(4n-5).\end{aligned}$$

□

Definition 2.1. *[23] The broom graph $B_{n,d}$ is a graph consisting of a path P_d, together with $(n-d)$ end vertices all adjacent to the same end vertex of P_d.*

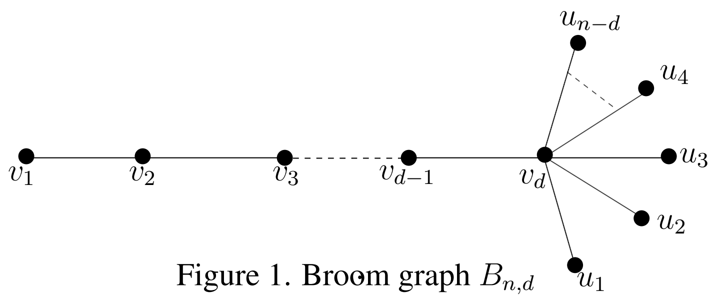

Figure 1. Broom graph $B_{n,d}$

Theorem 2.1. *The eccentric-distance sum of a broom graph $B_{n,d}$ is*

$$\xi^{ds}(P_{d+1}) + (n-d-1)\left[\frac{d}{2}(d^2-3d+4n) + \sum_{k=\frac{d}{2}}^{d} k^2 + \sum_{k=1}^{\frac{d}{2}-1}(d-k)k\right],$$

when d is even,

$$\xi^{ds}(P_{d+1}) + (n-d-1)\left[\frac{d}{2}(d^2-3d+4n) + \sum_{k=\frac{d-1}{2}+1}^{d} k^2 + \sum_{k=1}^{\frac{d-1}{2}}(d-k)k\right],$$

when d is odd.

Proof. Let $\{v_1, v_2, \ldots, v_d, u_1, u_2, \ldots, u_{n-d}\}$ be the set of n vertices of the broom graph $B_{n,d}$. We consider the following cases.

Case(i): d is even.
$e(v_1) = d$, $e(v_2) = e(v_d) = d-1$, $e(v_3) = e(v_{d-1}) = d-2$, $\ldots$, $e(v_{\frac{d}{2}}) = e(v_{\frac{d}{2}+2}) = \dfrac{d}{2}+1$, $e(v_{\frac{d}{2}+1}) = \dfrac{d}{2}$, and $e(u_1) = e(u_2) = \cdots = e(u_{n-d}) = d$. Then,

$$\begin{aligned}\xi^{ds}(B_{n,d}) =& \xi^{ds}(P_{d+1}) + \{[d+d]\,d + [(d-1)+d]\,(d-1) + [(d-2)+d]\,(d-2) + \cdots \\ &+ \left[\left(\frac{d}{2}+2\right)+d\right]\left(\frac{d}{2}+2\right) + \left[\left(\frac{d}{2}+1\right)+d\right]\left(\frac{d}{2}+1\right) + \left[\frac{d}{2}+d\right]\frac{d}{2} \\ &+ \left[\left(\frac{d}{2}+1\right)+d\right]\left(\frac{d}{2}-1\right) + \cdots + [(d-2)+d]\,2 + [(d-1)+d]\,1\Big\}\,(n-d-1) \\ &+ (d+d)\,2\,[(n-d-1)+(n-d-2)+\cdots+2+1] \\ =& \xi^{ds}(P_{d+1}) + (n-d-1)\left[\frac{d}{2}(d^2-3d+4n) + \sum_{k=\frac{d}{2}}^{d} k^2 + \sum_{k=1}^{\frac{d}{2}-1}(d-k)k\right].\end{aligned}$$

Case(ii): d is odd.
$e(v_1) = d$, $e(v_2) = e(v_d) = d-1$, $e(v_3) = e(v_{d-1}) = d-2, \ldots, e(v_{\frac{d+1}{2}-1}) = e(v_{\frac{d+1}{2}+2}) = \dfrac{d-1}{2}+2$, $e(v_{\frac{d+1}{2}}) = e(v_{\frac{d+1}{2}+1}) = \dfrac{d-1}{2}+1$, and $e(u_1) = e(u_2) = \cdots = e(u_{n-d}) = d$. Then,

$$\begin{aligned}\xi^{ds}(B_{n,d}) =& \xi^{ds}(P_{d+1}) + \{[d+d]\,d + [(d-1)+d]\,(d-1) + [(d-2)+d]\,(d-2) + \cdots \\ &+ \left[\left(\frac{d-1}{2}+2\right)+d\right]\left(\frac{d-1}{2}+2\right) + \left[\left(\frac{d-1}{2}+1\right)+d\right]\left(\frac{d-1}{2}+1\right) \\ &+ \left[\left(\frac{d-1}{2}+1\right)+d\right]\left(\frac{d-1}{2}\right) + \left[\left(\frac{d-1}{2}+2\right)+d\right]\left(\frac{d-1}{2}-1\right) + \cdots \\ &+ [(d-2)+d]\,2 + [(d-1)+d]\,1\}\,(n-d-1) + (d+d)\,2\,[(n-d-1) \\ &+ (n-d-2)+\cdots+2+1] \\ =& \xi^{ds}(P_{d+1}) + (n-d-1)\left[\frac{d}{2}(d^2-3d+4n) + \sum_{k=\frac{d-1}{2}+1}^{d} k^2 + \sum_{k=1}^{\frac{d-1}{2}}(d-k)k\right].\end{aligned}$$

$\square$

Definition 2.2. *[23] The lollipop graph $L_{n,d}$ is a graph obtained from a complete graph K_{n-d} and a path P_d, by joining one of the end vertices of P_d to all the vertices of K_{n-d}.*

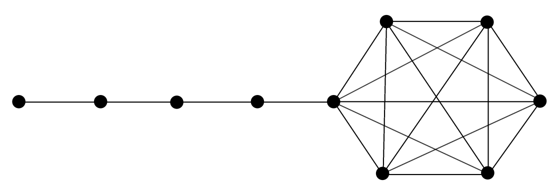

Figure 2. Lollipop graph $L_{10,5}$

Theorem 2.2. *The eccentric-distance sum of a lollipop graph $L_{n,d}$ is*

$$\xi^{ds}(P_{d+1}) + (n-d-1)\left[\frac{d^2}{2}(d+1) + d(n-d) + \sum_{k=\frac{d}{2}}^{d} k^2 + \sum_{k=1}^{\frac{d}{2}-1}(d-k)k\right],$$

when d is even,

$$\xi^{ds}(P_{d+1}) + (n-d-1)\left[\frac{d^2}{2}(d+1) + d(n-d) + \sum_{k=\frac{d-1}{2}+1}^{d} k^2 + \sum_{k=1}^{\frac{d-1}{2}}(d-k)k\right],$$

when d is odd.

Proof. Let $\{v_1, v_2, \ldots, v_d, u_1, u_2, \ldots, u_{n-d}\}$ be the set of n vertices of the lollipop graph $L_{n,d}$. We consider the following cases.

Case(i): d is even.

$$\begin{aligned}
\xi^{ds}(L_{n,d}) =& \xi^{ds}(P_{d+1}) + \{[d+d]\,d + [(d-1)+d]\,(d-1) + [(d-2)+d]\,(d-2) + \cdots \\
&+ \left[\left(\frac{d}{2}+2\right)+d\right]\left(\frac{d}{2}+2\right) + \left[\left(\frac{d}{2}+1\right)+d\right]\left(\frac{d}{2}+1\right) + \left[\frac{d}{2}+d\right]\frac{d}{2} \\
&+ \left[\left(\frac{d}{2}+1\right)+d\right]\left(\frac{d}{2}-1\right) + \cdots + [(d-2)+d]\,2 + [(d-1)+d]\,1\Big\}\,(n-d-1) \\
&+ d\,(n-d-1)\,(n-d) \\
=& \xi^{ds}(P_{d+1}) + (n-d-1)\left[\frac{d^2}{2}(d+1) + d(n-d) + \sum_{k=\frac{d}{2}}^{d} k^2 + \sum_{k=1}^{\frac{d}{2}-1}(d-k)k\right].
\end{aligned}$$

Case(ii): d is odd.

$$\begin{aligned}
\xi^{ds}(L_{n,d}) =& \xi^{ds}(P_{d+1}) + \{[d+d]\,d + [(d-1)+d]\,(d-1) + [(d-2)+d]\,(d-2) + \cdots \\
&+ \left[\left(\frac{d-1}{2}+2\right)+d\right]\left(\frac{d-1}{2}+2\right) + \left[\left(\frac{d-1}{2}+1\right)+d\right]\left(\frac{d-1}{2}+1\right) \\
&+ \left[\left(\frac{d-1}{2}+1\right)+d\right]\left(\frac{d-1}{2}\right) + \left[\left(\frac{d-1}{2}+2\right)+d\right]\left(\frac{d-1}{2}-1\right) + \cdots \\
&+ [(d-2)+d]\,2 + [(d-1)+d]\,1\}\,(n-d-1) + d\,(n-d-1)\,(n-d) \\
=& \xi^{ds}(P_{d+1}) + (n-d-1)\left[\frac{d^2}{2}(d+1) + d(n-d) + \sum_{k=\frac{d-1}{2}+1}^{d} k^2 + \sum_{k=1}^{\frac{d-1}{2}}(d-k)k\right].
\end{aligned}$$

□

Definition 2.3. *[21] A double star graph $S_{n,m}$ is a graph constructed from $K_{1,n-1}$ and $K_{1,m-1}$ by joining their centers v_0 and u_0. The vertex-set $V(S_{n,m})$ is $V(K_{1,n-1}) \cup V(K_{1,m-1}) = \{v_0, v_1, \ldots, v_{n-1}, u_0, u_1, \ldots, u_{m-1}\}$ and the edge-set $E(S_{n,m}) = \{v_0u_0, v_0v_i, u_0u_j | 1 \leq i \leq (n-1); 1 \leq j \leq (m-1)\}$. Therefore, a double star graph is bipartite.*

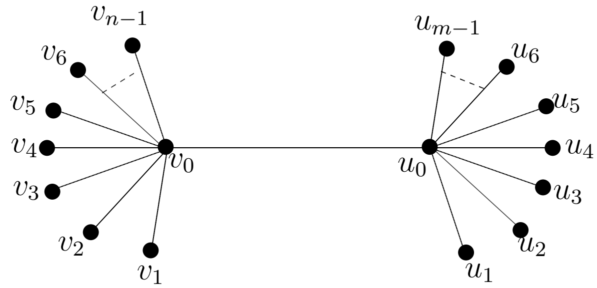

Figure 3. Double star graph $S_{n,m}$.

Theorem 2.3. *The eccentric-distance sum of a double star graph $S_{n,m}$ is $3[(n-2)(2n-1) + m(2m+5) + 6(n-1)(m-1) - 5]$.*

Proof. $e(v_0) = 2 = e(u_0)$ and $e(v_i) = 3 = e(u_j)$ where, $i = 1, 2, \ldots, n-1$ and $j = 1, 2, \ldots, m-1$ Then,

$$\begin{aligned}\xi^{ds}(S_{n,m}) =& [(3+2)1 + (3+2)2 + (3+3)3(m-1)](n-1) \\ &+ (2+2)1 + (2+3)2(m-1) + (2+3)1(m-1) \\ &+ (3+3)2(n-2) + (3+3)2(n-3) + \cdots + (3+3)2(1) \\ &+ (3+3)2(m-2) + (3+3)2(m-3) + \cdots + (3+3)2(1) \\ =& 3[(n-2)(2n-1) + m(2m+5) + 6(n-1)(m-1) - 5].\end{aligned}$$

□

Definition 2.4. *[13] The friendship (or Dutch windmill or fan) graph F_n is a graph constructed by joining n copies of the cycle graph C_3 with a common vertex.*

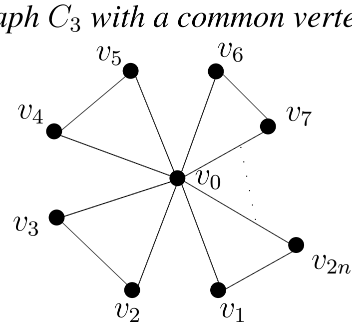

Figure 4. Friendship graph F_n

Theorem 2.4. *The eccentric-distance sum of friendship graph F_n is $2n(8n-3)$.*

Proof. $e(v_0) = 1$ and $e(v_i) = 2$ where, $i = 1, 2, \ldots, 2n$. Then,

$$\begin{aligned}
\xi^{ds}(F_n) =& (2+2)1 + (2+2)2(2n-2) + (2+2)2(2n-2) \\
& + (2+2)1 + (2+2)2(2n-4) + (2+2)2(2n-4) \\
& + (2+2)1 + (2+2)2(2n-6) + (2+2)2(2n-6) \\
& \vdots \\
& + (2+2)1 + (2+2)2(2) + (2+2)2(2) \\
& + (2+2)1 + (1+2)1(2n) \\
=& 2n(8n-3).
\end{aligned}$$

□

Definition 2.5. *[24] Consider the star graph $K_{1,n}$ with vertex set $\{v_0, v_1, v_2, \ldots, v_n\}$, introduce an edge to each of the pendant vertices $v_1, v_2, \ldots, v_n$ to get the resulting graph $K_{1,n,n}$ with vertices $\{v_0, v_1, \ldots, v_n, v_{n+1}, \ldots, v_{2n}\}$, again introduce an edge to each of the pendant vertices $v_{n+1}, \ldots, v_{2n}$, to get the graph $K_{1,n,n,n}$. Repeating this $(m-1)$ times we get a graph $K_{1,\underbrace{n, n, \ldots, n}_{m-times}}$ called multi-star graph with $(mn+1)$ vertices $v_0, v_1, v_2, \ldots, v_n$, $v_{n+1}, \ldots$, v_{2n}, $v_{2n+1}, \ldots, v_{3n}, \ldots$, $v_{(m-1)n+1}, \ldots, v_{mn}$ and mn edges, as shown in Figure 5.*

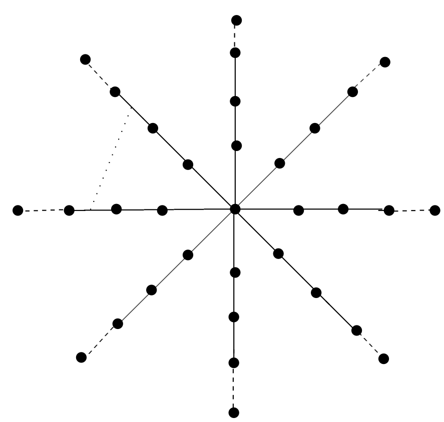

Figure 5. Multi-star graph $K_{1,\underbrace{n, n, \ldots, n}_{m-times}}$

Theorem 2.5. *The eccentric-distance sum of a multi-star graph $K_{1,n,n,\ldots,n}$ is*

$$\frac{nm}{6}(m+1)(8m+1) + n(n+1)\left\{ m \left[2\sum_{k=1}^{m}(k+1)k + \sum_{k=1}^{m-1}(m+1+k)(m-k) \right]\right.$$

$$+\sum_{k=1}^{m}k+\frac{1}{2}\left[\sum_{k=1}^{m}k(k+1)^2+\sum_{k=1}^{m-1}(k+1)(k+2)\right]+1\Bigg\}+\frac{n}{2}\left[\sum_{k=1}^{m-1}(m+k+1)k(k+1)\right.$$
$$\left.+\sum_{k=1}^{m-1}(m+k)(m-k)(m-k+1)\right].$$

Proof.

$$\begin{aligned}
&\xi^{ds}\left(K_{1,\underbrace{n,n,\ldots,n}_{m-times}}\right)\\
&=\{[m+(m+1)]1+[m+(m+2)]2+\cdots+[m+(m+m)]m\}n\\
&\quad+\{[(m+1)+(m+1)]2+[(m+1)+(m+2)]3+\cdots+[(m+1)+(m+m)](m+1)\}\\
&\quad[(n-1)+(n-2)+(n-3)+\cdots+2+1]\\
&\quad+\{[(m+2)+(m+1)]3+[(m+2)+(m+2)]4+\cdots+[(m+2)+(m+m)](m+2)\}\\
&\quad[(n-1)+(n-2)+(n-3)+\cdots+2+1]\\
&\quad\vdots\\
&\quad+\{[(m+m)+(m+1)](m+1)+[(m+m)+(m+2)](m+2)+\cdots\\
&\quad+[(m+m)+(m+m)](m+m)\}[(n-1)+(n-2)+(n-3)+\cdots+2+1]\\
&\quad+\{[(m+1)+(m+2)]1+[(m+1)+(m+3)]2+\cdots+[(m+1)+(m+m)](m-1)\}n\\
&\quad+\{[(m+2)+(m+3)]1+[(m+2)+(m+4)]2+\cdots+[(m+2)+(m+m)](m-2)\}n\\
&\quad\vdots\\
&\quad+\{[(m+(m-1))+(m+m)]1\}n
\end{aligned}$$

$$\begin{aligned}
&=\frac{nm}{6}(m+1)(8m+1)+n(n+1)\Bigg\{m\left[2\sum_{k=1}^{m}(k+1)k+\sum_{k=1}^{m-1}(m+1+k)(m-k)\right]\\
&\quad+\sum_{k=1}^{m}k+\frac{1}{2}\left[\sum_{k=1}^{m}k(k+1)^2+\sum_{k=1}^{m-1}(k+1)(k+2)\right]+1\Bigg\}\\
&\quad+\frac{n}{2}\left[\sum_{k=1}^{m-1}(m+k+1)k(k+1)+\sum_{k=1}^{m-1}(m+k)(m-k)(m-k+1)\right].
\end{aligned}$$

□

Definition 2.6. *[24] Pl_n $(n\geq 3)$ is a graph obtained by the join of P_{n-2} and P_2, as shown in Figure 6.*

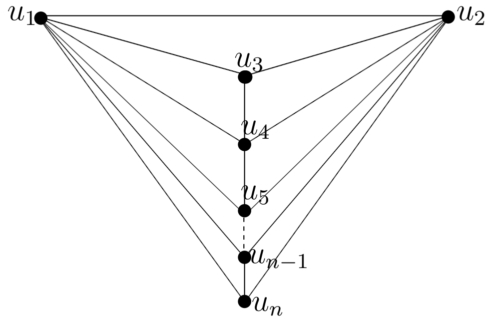

Figure 6. Pl_n graph, $n \geq 3$.

Theorem 2.6. *The eccentric-distance sum of* Pl_n $(n \geq 6)$ *graph is* $4n^2 + 19n - 29$.

Proof. $e(u_1) = 1 = e(u_2)$ and $e(u_i) = 2$ where, $i = 3, 4, \ldots, n$. Then,

$$\begin{aligned}\xi^{ds}(Pl_n) =&(1+2)1(n-2) + (1+1)1 + (1+2)1(n-2)\\ &+ (2+2)1 + (2+2)2(n-4)\\ &+ (2+2)1 + (2+2)2(n-5)\\ &\vdots\\ &+ (2+2)1 + (2+2)2(1)\\ &+ (2+2)1\\ =&4n^2 + 19n - 29.\end{aligned}$$

□

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References

[1] A.R. Ashrafi, T. Došlić and M. Saheli, The eccentric connectivity index of TUC_4C_8(R) nanotubes, *MATCH Commun. Math. Comput. Chem.* **65** (2011), 221–230.

[2] A.R. Ashrafi, M. Saheli and M. Ghorbani, The eccentric connectivity index of nanotubes and nanotori, *J. Comput. Appl. Math.* **235** (2011), 4561–4566.

[3] M. Azari and A. Iranmanesh, Computing the eccentric-distance sum for graph operations, *Discrete Appl. Math.* **161** (2013), 2827-2840.

[4] A. Dobrynin, R. Entringer and I. Gutman, Wiener index of trees: theory and applications, *Acta Appl. Math.* **66** (2001), 211–249.

[5] A. Dobrynin, I. Gutman, S. Klavžar and P. Žigert, Wiener index of hexagonal systems, *Acta Appl. Math.* **72** (2002), 247–294.

[6] A. Dobrynin and A.A. Kochetova, Degree distance of a graph: A degree analogue of the Wiener index, *J. Chem. Inf. Comput. Sci.* **34** (1994), 1082–1086.

[7] T. Došlić and M. Saheli, Eccentric connectivity index of composite graphs, manuscript, (2009).

[8] T. Došlić, M. Saheli and D. Vukičević, Eccentric connectivity index: Extremal graphs and values, *Iran. J. Math. Chem.* **1** (2010), 45–56.

[9] H. Dureja, S. Gupta and A.K. Madan, Predicting anti-HIV-1 activity of 6-arylbenzonitriles: Computational approach using superaugmented eccentric connectivity topochemical indices, *J. Mol. Graph. Model.* **26** (2008), 1020–1029.

[10] J.A. Gallian, Dynamic Survey DS6: Graph Labeling, *Electronic J. Combinatorics* (2007), 1–58.

[11] S. Gupta, M. Singh and A.K. Madan, Application of graph theory: Relationship of eccentric connectivity index and Wieners index with anti-inflammatory activity, *J. Math. Anal. Appl.* **266** (2002), 259–268.

[12] S. Gupta, M. Singh and A.K. Madan, Eccentric-distance sum: A novel graph invariant for predicting biological and physical properties, *J. Math. Anal. Appl.* **275** (2002), 386–401.

[13] I. Gutman, Selected properties of the Schultz molecular topological index, *J. Chem. Inf. Comput. Sci.* **34** (1994), 1087–1089.

[14] F. Harary, *Graph Theory*, Addison Wesley, Reading Mass (1969).

[15] H. Hua, Wiener and Schultz molecular topological indices of graphs with specified cut edges, *MATCH Commun. Math. Comput. Chem.* **61** (2009), 643–651.

[16] H.B. Hua, K.X. Xu and W.N. Shu, A short and unified proof of Yu et al.s two results on the eccentric-distance sum, *J. Math. Anal. Appl.* **382** (2011), 364–366.

[17] A. Ilić, Eccentric connectivity index, in: I. Gutman, B. Furtula (Eds.), Novel Molecular Structure Descriptors Theory and Applications II, in: *Math. Chem. Monogr.* **9** University of Kragujevac (2010).

[18] A. Ilić and I. Gutman, Eccentric connectivity index of chemical trees, *MATCH Commun. Math. Comput. Chem.* **65** (2011), 731–744.

[19] A. Ilić, S. Klavžar and D. Stevanović, Calculating the degree distance of partial hamming graphs, *MATCH Commun. Math. Comput. Chem.* **63** (2010), 411–424.

[20] A. Ilić, G. Yu and L. Feng, On the eccentric-distance sum of graphs, *J. Math. Anal. Appl.* **381** (2011), 590–600.

[21] D. Indriati, Widodo, I. E. Wijayanti and K. A. Sugeng, On total irregularity strength of double-star and related graphs, *Procedia Computer Science* **74** (2015), 118–123.

[22] V. Kumar, S. Sardana and A.K. Madan, Predicting anti-HIV activity of 2, 3-diaryl-1, 3 thiazolidin-4-ones: Computational approach using reformed eccentric connectivity index, *J. Mol. Model.* **10** (2004), 399–407.

[23] M.J. Morgan, S. Mukwembi, H.C. Swart, On the eccentric connectivity index of a graph, *Discrete Math.* **311** (2011), 1229–1234.

[24] S. Ramakrishnan and J. B. Babujee, Degree Distance of Some Planar Graphs, *International Journal of Computing Algorithm,* **03** (2014), 541–544.

[25] S. Sardana and A.K. Madan, Predicting anti-HIV activity of TIBO derivatives: a computational approach using a novel topological descriptor, *J. Mol. Model.* **8** (2002), 258–265.

[26] V. Sharma, R. Goswami, A.K. Madan, Eccentric connectivity index: A novel highly discriminating topological descriptor for structure property and structure activity studies, *J. Chem. Inf. Comput. Sci.* **37** (1997), 273–282.

[27] G. Yu, L. Feng and A. Ilić, On the eccentric-distance sum of trees and unicyclic graphs, *J. Math. Anal. Appl.* **375** (2011), 99–107.

[28] B. Zhou and Z. Du, On eccentric connectivity index, *MATCH Commun. Math. Comput. Chem.* **63** (2010), 181–198.