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Antimagicness for a family of generalized antiprism graphs

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Abstract

An antimagic labeling of a graph $G = (V, E)$ is a bijection from the set of edges E to the set of integers $\{1, 2, \dots, |E|\}$ such that all vertex weights are pairwise distinct, where the weight of a vertex is the sum of all edge labels incident with that vertex. A graph is antimagic if it has an antimagic labeling. In this paper we provide constructions of antimagic labelings for a family of generalized antiprism graphs and generalized toroidal antiprism graphs.

Keywords: antimagic labeling, antimagic generalized antiprism graph, antimagic generalized toroidal antiprism graph
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1. Introduction

All graphs considered in this paper are finite, simple, undirected and connected. A *labeling* of a graph $G = (V, E)$ is a bijection from some set of graph elements to a set of numbers. In particular, in this paper we are interested in labeling of the edges of a graph. A labeling $l : E \longrightarrow \{1, 2, \ldots, |E|\}$ is called an *edge labeling*. The *weight* of a vertex v is defined by $wt(v) = \sum_{u \in N(v)} l(uv)$, where $N(v)$ is the set of the neighbors of v. An edge labeling l of G is *antimagic* if all vertex weights in G are pairwise distinct. A graph G is *antimagic* if it has an antimagic labeling.

Hartsfield and Ringel [6] showed that path P_m, star S_m, cycle C_m, complete graph K_m, wheel W_m and bipartite graph $K_{2,m}$, $m \geq 3$, are antimagic. They conjectured that every connected graph other than K_2 is antimagic. Over the period of more than two decades, many families of graphs have been proved to be antimagic, for example, see [1, 3, 5, 6, 9, 10, 11]. However, the general conjecture is not yet settled. Even the weaker conjecture "Every tree different from K_2 is antimagic" still remains open. The results concerning antimagic labeling of graphs are summarized in [5], see also [4].

In 1969, Dickson [2] introduced completely separating system. A *completely separating system* (CSS) on a finite set $[n] = \{1, 2, \ldots, n\}$ (or (n)CSS) is a collection of subsets of $[n]$ in which for each pair of elements $a \neq b \in [n]$, there exist two subsets A and B of $[n]$ in
$mathcalC$ such that A contains a but not b and B contains b but not a. A d-*element* in a collection of sets is an element which occurs in exactly d sets in the collection. If $|A| = k$, for all $A \in \mathcal{C}$, then $\mathcal{C}$ is said to be an (n, k)CSS. For example, the collection $\{\{1, 2\}, \{1, 3\}\}$ is not a $(3, 2)$CSS, while the collection $\{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$ is a $(3, 2)$CSS. For any n, k fixed positive integers, $R(n, k) = \min\{|\mathcal{C}| : \mathcal{C} \text{ is an } (n, k)\text{CSS}\}$. An (n, k)CSS for which $|\mathcal{C}| = R(n, k)$ is a *minimal* (n, k)CSS.

Roberts [8], among others, has explored minimal (n, k)CSS and gave a method for the construction of minimal (n, k)CSSs. In the next section we review a relationship between CSSs and antimagic labeling of graphs.

2. Preliminaries

In this section we recall a result from [9], that is, a construction of antimagic labeling of regular graphs that uses a relationship between CSSs and edge labelings of graphs, coupled with Roberts' construction [8].

We next describe the construction.

Roberts' construction [8]

Assume that $k \geq 2$, $n \geq \binom{k+1}{2}$ and $k|2n$, and let $R = R(n, k) = 2n/k$. An $(R \times k)$-array L is constructed, where each row of L forms a subset of $[n]$ and the R rows of L form an (n, k)CSS. Let e_{ij} denote the element of L in row i and column j. Initialize all elements of L to zero. For e

from 1 to n, in order, include e in the two positions of L defined by

$$\min_j \min_i \{e_{ij} : e_{ij} = 0\},$$
$$\min_i \min_j \{e_{ij} : e_{ij} = 0\}.$$

That is, e is placed in the first row of L containing a 0, in the first 0-valued place in that row, e is then also placed in the first column of L containing a 0, in the first 0-valued place in that column. Each of the integers 1 to n appears in L in two positions, and the array L is the array of an (n, k)CSS. This concludes Roberts' construction.

The following theorems will be useful when creating antimagic labelings of graphs in the family of generalized antiprism graphs.

Theorem 2.1. *[9] Let $V = \{v_1, \ldots, v_p\}$ be a collection of subsets of $[q]$. If V is a (q)CSS in which each element of $[q]$ is a 2-element and E is the set of all unordered pairs $\{v_i, v_j\}$, where $v_i \cap v_j \neq \emptyset$, then $G = (V, E)$ is a simple graph, $|V| = p$ and $|E| = q$. Also, G has an edge labeling l given by $l(v_i, v_j) = v_i \cap v_j$.*

Theorem 2.2. *[9] Let $G = (V, E)$ be a simple graph with $|V| = p$, $|E| = q$ with an edge labeling given by bijection $l : E \to [q]$. For $v \in V$, let S_v be the set of labels of edges incident with v. Then the collection $\{S_v \mid v \in V\}$ is a (q)CSS consisting of 2-elements.*

Note that if $V = \{v_1, \ldots, v_p\}$ is a (q, k)CSS then G is a k-regular graph together with an edge labeling and vice versa.

An edge labeling of a graph will be represented by an array, not necessary rectangular, in which each vertex is represented by a row and each row consists of the labels of all edges incident with the vertex represented by that row.

Theorem 2.3. *[9] Let L be the array of a (q, k)CSS obtained using Roberts' construction. Then the k-regular graph $G(V, E)$, where $|V| = p = 2q/k$ and $|E| = q$, has an antimagic edge labeling L.*

We next illustrate Roberts' construction by using it to create a $(6, 3)$CSS and its corresponding antimagic labeling of the 3-regular graph with 4 vertices in Figure 1.

1	2	3
1	4	5
2	4	6
3	5	6

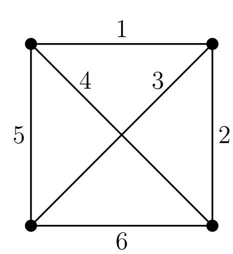

Figure 1. The $(6, 3)$CSS obtained using Roberts' construction and the corresponding graph K_4 with antimagic edge labeling.

We conclude this section with definitions of some families of graphs that will be used in this paper.

To start with, based on the definition of generalized antiprism graph from [4], we extend the concept to a more general one. Let G be any regular graph with m vertices. A *generalized antiprism* graph A^n_G is a graph obtained by completing the generalized prism graph $G \times P_n$, $m \geq 3$ and $n \geq 2$, by edges $\{v_{i,j+1}v_{i+1,j} : 1 \leq i \leq m-1, 1 \leq j \leq n-1\} \cup \{v_{m,j+1}v_{1,j} : 1 \leq j \leq n-1\}$. That is, the vertex set of A^n_G is $V(A^n_G) = V(G \times P_n) = \{v_{i,j} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and the edge set of A^n_G is $E(A^n_G) = E(G \times P_n) \cup \{v_{i,j+1}v_{i+1,j} : 1 \leq i \leq m, 1 \leq j \leq n-1\}$, where i is taken modulo m.

The generalized antiprism graph A^n_m in [4] is a special case of A^n_G when $G = C_m$. Throughout this section we use $A^n_{C_m}$ instead of A^n_m. A copy of G in A^n_G is called a *layer* of A^n_G. An *outer layer* is a layer that contains all vertices with degree $d-2$ while each vertex in each *inner layer* has degree d, for example, see Figure 2.

A *generalized antiprism balloon* B^n_G obtained from the generalized antiprism A^n_G by connecting each vertex of one outer layer of A^n_G to an external vertex and each vertex of the other outer layer to an another external vertex. In particular, $B^n_{C_m}$ is called a *generalized antiprism $2mn$-hedron balloon*.

A *generalized antiprism tower* TW^n_G is obtained from B^n_G by deleting an external vertex connected to each vertex of the outer layer of A^n_G.

A *generalized toroidal antiprism* graph T^n_G is a graph obtained from the generalized antiprism graph A^n_G by joining the two outer layers of the generalized antiprism graph with the edges in the same way as joining between two consecutive layers of the generalized antiprism graph, see Figure 3 as an example.

3. Results

Theorem 3.1. *Let G be any antimagic C_m or K_m, $m \geq 3$, obtained by Roberts' construction. Then the generalized antiprism graph A^n_G, $n \geq 2$, is antimagic.*

Proof. Assume that G has m vertices and q edges. Let L_j, $1 \leq j \leq n$, be the array of the edge labels of G_j, where G_j is the j-th copy of G in A^n_G, $n \geq 2$. Let T_l, $1 \leq l \leq 2(n-1)$, be the $(m \times 1)$-array of edges e^l_i, $1 \leq i \leq m$, where e^l_i are the edges of A^n_G that do not belong to any copy G_j. We construct the array A of edge labels of A^n_G, $n \geq 2$, as follows.

(1) Replace the edge labels in the array L_j, $1 \leq j \leq n$, with new labels by adding $2(j-1)m + (j-1)q$ to each of the original edge labels;
(2) Label the edge e^l_i, $1 \leq i \leq m$, in row i of the array T_l, $1 \leq l \leq 2(n-1)$, with $\left\lceil \frac{l}{2} \right\rceil q + (l-1)m + 2i - 1$, for $l \equiv 1 \, mod \, 2$, and $\frac{l}{2}q + (l-2)m + 2i$, for $l \equiv 0 \, mod \, 2$;
(3) Form the array A as shown below.
For $n = 2$,

$$\begin{matrix} L_1 & T_1 & T_2 \\ T^*_1 & T^*_2 & L_2 \end{matrix}$$

for $n = 3$,

$$\begin{array}{ccccc} & & L_1 & T_1 & T_2 \\ & & L_2 & T_3 & T_4 \\ T_1^* & T_2^* & T_3^* & T_4^* & L_3 \end{array}$$

and for $n \geq 4$,

$$\begin{array}{ccccc} & & L_1 & T_1 & T_2 \\ & & L_2 & T_3 & T_4 \\ T_1^* & T_2^* & L_3 & T_5 & T_6 \\ T_3^* & T_4^* & L_4 & T_7 & T_8 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ T_{2(n-3)-1}^* & T_{2(n-3)}^* & L_{n-1} & T_{2(n-1)-1} & T_{2(n-1)} \\ T_{2(n-2)-1}^* & T_{2(n-2)}^* & T_{2(n-1)-1}^* & T_{2(n-1)}^* & L_n \end{array}$$

where $T_l^* = (e_1^l \ e_1^{l+1} \ e_2^{l+1} \dots e_{m-2}^{l+1} \ e_{m-1}^{l+1})^t$ and $T_{l+1}^* = (e_2^l \ e_3^l \ e_4^l \dots e_m^l \ e_m^{l+1})^t$, for $l \equiv 1 \ mod \ 2$ (see, for example, the array of edge labels in Figure 2).

By the construction of the array A, it is clear that the weight of each vertex (row) in the array is less than the weight of the vertex (row) below. □

We illustrate the generalized antiprism graph $A_{C_4}^3$ with antimagic labeling in Figure 2.

		1	2	5	6
		1	3	7	8
		2	4	9	10
		3	4	11	12
		13	14	17	18
		13	15	19	20
		14	16	21	22
		15	16	23	24
5	7	17	19	25	26
6	9	18	21	25	27
8	11	20	23	26	28
10	12	22	24	27	28

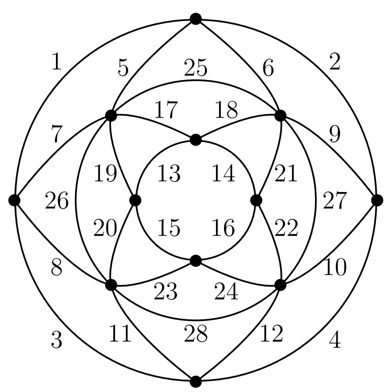

Figure 2. The generalized antiprism graph $A_{C_4}^3$ with antimagic labeling.

Corollary 3.1. (i) *The generalized antiprism* $4m$*-hedron balloon graph* $B_{C_m}^2$*,* $m \geq 3$*, is antimagic.*

(ii) *The generalized antiprism* $2mn$*-hedron balloon graph* $B_{C_m}^n$*,* $3 \leq m \leq 6$ *and* $n \geq 3$*, is antimagic.*

Proof. (i) Let S_f, $1 \leq f \leq 2$, be the array $(m \times 1)$-array of edges e_i, $1 \leq i \leq m$, where e_i are the edges of $B_{C_m}^2$ that do not belong to $A_{C_m}^2$. We consider two cases.

Case 1: $3 \leq m \leq 4$

We construct the array B of edge labels of B_G^2 as follows.

(1) Label the edges e_i, $1 \le i \le m$, in the row i of the array S_f, $1 \le f \le 2$, with $i + (f-1)m$;
(2) Replace the edge labels in the array A of the construction as given in the proof of Theorem 3.1 with new labels by adding $2m$ to each of the original edge labels of A;
(3) Form the array B as shown below.

$$\begin{array}{cccc} & & & S_1^t \\ & & & S_2^t \\ S_1 & L_1 & T_1 & T_2 \\ S_2 & T_1^* & T_2^* & L_2 \end{array}$$

Case 2: $m \ge 5$

(1) Keep the array A of the construction as given in the proof of Theorem 3.1;
(2) Label the edges e_i, $1 \le i \le m$, in the row i of the array S_f, $1 \le f \le 2$, with $i + (f+3)m$;
(3) Form the array B as shown below.

$$\begin{array}{cccc} L_1 & T_1 & T_2 & S_1 \\ T_1^* & T_2^* & L_2 & S_2 \\ & & & S_1^t \\ & & & S_2^t \end{array}$$

By the construction of the array B, in both cases it is clear that the weight of each vertex (row) in the array is less than the weight of the vertex (row) below with two exceptions. These are the weights of the last row of the subarray $T_1^* T_2^* L_2 S_2$ and the array S_1^t in Case 2 that need to be verified.

Let $e_{g,h}$ be the label at the row g and column h in the array B. Let r_{2m} be the last row of $T_1^* T_2^* L_2 S_2$ and $r_{2m+1} = S_1^t$. We have the labels in the rows r_{2m} and r_{2m+1} as shown.

$$\begin{array}{lcccc} r_{2m}: & \dots & e_{2m,m-2} & e_{2m,m-1} & e_{2m,m} \\ r_{2m+1}: & \dots & e_{2m+1,m-2} & e_{2m+1,m-1} & e_{2m+1,m} \end{array}$$

Since $\Sigma_{h=m-2}^{m} e_{2m,h} = 14m - 1 < 15m - 3 = \Sigma_{h=m-2}^{m} e_{2m+1,h}$ and $e_{2m,h} < e_{2m+1,h}$, for $m - 4 \le h \le m - 3$, hence $wt(r_{2m}) < wt(r_{2m+1})$.

(ii) Let S_f, $1 \le f \le 2$, be the array $(m \times 1)$-array of edges e_i, $1 \le i \le m$, where e_i are the edges of $B_{C_m}^n$ that do not belong to $A_{C_m}^n$, $n \ge 3$ and $3 \le m \le 6$. We construct the array B of edge labels of $B_{C_m}^n$, $n \ge 3$ and $3 \le m \le 6$, as follows.

(1) Label the edge e_i, $1 \le i \le m$, in the row i of the array S_f, $1 \le f \le 2$, with $i + (f-1)m$;
(2) Replace the edge labels in the array A of the construction as given in the proof of Theorem 3.1 with new labels by adding $2m$ to each of the original edge labels of A;

(3) Form the array B as shown below.

$$\begin{array}{ccccc}
 & & & & S_1^t \\
 & & & & S_2^t \\
 & S_1 & L_1 & T_1 & T_2 \\
 & S_2 & L_2 & T_3 & T_4 \\
T_1^* & T_2^* & L_3 & T_5 & T_6 \\
T_3^* & T_4^* & L_4 & T_7 & T_8 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
T^*_{2(n-3)-1} & T^*_{2(n-3)} & L_{n-1} & T_{2(n-1)-1} & T_{2(n-1)} \\
T^*_{2(n-2)-1} & T^*_{2(n-2)} & T^*_{2(n-1)-1} & T^*_{2(n-1)} & L_n
\end{array}$$

By the construction of the array B, it is clear that the weight of each vertex (row) in the array is less than the weight of the vertex (row) below with two exceptions. These are the weights of the row S_2^t and the first row (r_3) of the subarray $S_1L_1T_1T_2$ that need to be verified.

We have $wt(S_2^t) = \frac{3m^2+m}{2} < 10m + 7 = wt(r_3)$, for $3 \leq m \leq 6$. □

Corollary 3.2. *The generalized* $2mn$*-hedron balloon graph* $B^n_{K_m}$, $m \geq 3$ *and* $n \geq 2$, *is antimagic.*

Proof. The proof follows immediately when C_m is replaced by K_m in the construction of the proof of Corollary 3.1 (ii), so it is omitted. □

Corollary 3.3. (i) *The generalized antiprism tower graph* $TW^2_{C_m}$, $m \geq 3$, *is antimagic.*
(ii) *The generalized antiprism tower graph* $TW^n_{C_m}$, $3 \leq m \leq 11$ *and* $n \geq 3$, *is antimagic.*

Proof. The proof follows immediately by deleting the arrays S_1 and S_1^t from the proof of Corollary 3.1 and reducing each entry of the resulting array by m. Moreover, for (ii) the first row of the subarray $L_1T_1T_2$ is the second row (r_2) of the entire array of the edge labels. We have $wt(S_2^t) = \frac{m(m+1)}{2} < 6m + 6 = wt(r_2)$, for $3 \leq m \leq 11$. □

Corollary 3.4. *The generalized antiprism tower graph* $TW^n_{K_m}$, $m \geq 3$ *and* $n \geq 2$, *is antimagic.*

Proof. The proof follows immediately when C_m is replaced by K_m in the construction of the proof of Corollary 3.1 (ii), and deleting the array S_1 and S_1^t from the construction. Finally, we reduce each entry of the resulting array by m. □

Recall that Theorem 3.1 gives antimagicness for every generalized antiprism graph A^n_G, for $G = C_m, K_m$, for $m \geq 3$ and $n \geq 2$. We can extend this to a further result of antimagicness for generalized toroidal antiprism graphs.

Theorem 3.2. *Let* G *be either an antimagic graph* C_m *or* K_m, $m \geq 3$, *obtained by Roberts' construction. Then, for* $n \geq 3$, *the generalized toroidal antiprism graph* T^n_G *is antimagic.*

Proof. Assume that G has $m \geq 3$ vertices and q edges. Let L_j, $1 \leq j \leq n$, be the array of edge labels of the j-th copy of G in T^n_G, for $n \geq 3$. Let T_l, $1 \leq l \leq 2n$, be the $(m \times 1)$-array of edges e^l_i, $1 \leq i \leq m$, where e^l_i are the edges of T^n_G that do not belong to any copy of G. We construct the array A of the edge labels of T^n_G, for $n \geq 3$. We consider two cases.

Case 1: n even

(1) Label the edge e_i^l, $1 \leq i \leq m$, in row i of the array T_l, $1 \leq l \leq 2n$, with $(\lceil \frac{l}{2} \rceil - 1)q + (l-1)m + 2i - 1$, for $l \equiv 1 \ mod \ 2$, and $(\frac{l}{2} - 1)q + (l-2)m + 2i$, for $l \equiv 0 \ mod \ 2$;
(2) Replace the edge labels in the array L_j, $1 \leq j \leq n$, with new labels by adding $2jm + (j-1)q$ to each of the original edge labels;
(3) Form the array A as shown below.

$$\begin{array}{ccccc} T_1 & T_2 & L_1 & T_3 & T_4 \\ T_1^* & T_2^* & L_2 & T_5 & T_6 \\ T_3^* & T_4^* & L_3 & T_7 & T_8 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ T_{2n-5}^* & T_{2n-4}^* & L_{n-1} & T_{2n-1} & T_{2n} \\ T_{2(n-1)-1}^* & T_{2(n-1)}^* & T_{2n-1}^* & T_{2n}^* & L_n \end{array}$$

By the construction of the array A, it is clear that the weight of each vertex (row) is less than the weight of the vertex (row) below with some exceptions. These are the weights of the last row (r_m) and the first row (r_{m+1}) of the subarrays $T_1T_2L_1T_3T_4$ and $T_1^*T_2^*L_2T_5T_6$, respectively, that need to be verified.

Let $e_{g,h}$ be the edge label at row g and column h in the array A.

We first consider $G = C_m$. In this case, we have the edge labels in rows r_m and r_{m+1} as shown below.

$$\begin{array}{lcccccc} r_m: & 2m-1 & 2m & \ldots & q+2m & q+4m-1 & q+4m \\ r_{m+1}: & 1 & 3 & \ldots & q+4m+2 & 2q+4m+1 & 2q+4m+2 \end{array}$$

Since $e_{m,1} + e_{m,2} + e_{m,4} + e_{m,5} + e_{m,6} = 3q + 14m - 2 < 5q + 13m + 6 = e_{m+1,1} + e_{m+1,2} + e_{m+1,4} + e_{m+1,5} + e_{m+1,6}$ and $e_{m,3} < e_{m+1,3}$, hence $wt(r_m) < wt(r_{m+1})$. It follows immediately when $G = K_m$.

Cases 2: n odd

The construction of Case 1 cannot provide the antiprism property when n is odd. However, we can modify the second subarray $T_1^*T_2^*L_2T_5T_6$ of the construction to meet that property. Let E_h, $1 \leq h \leq m$, be row h of $T_1^*T_2^*$ in the subarray $T_1^*T_2^*L_2T_5T_6$, that is, $E_1 = (1\ 3)$, $E_h = (2 + 2(h-2)\ \ 5 + 2(h-2))$, for $2 \leq h \leq m-1$, and $E_h = (2 + 2(h-2)\ \ 5 + 2(h-2) - 1)$, for $h = m$. When $m \equiv 0 \ mod \ 2$, we swap E_2 and E_3, E_4 and E_5, $\ldots$, E_{m-2} and E_{m-1}, (resp., when $m \equiv 1 \ mod \ 2$, we swap E_2 and E_3, E_4 and E_5, $\ldots$, E_{m-1} and E_m). Then we have the resulting subarray $E^*L_2T_5T_6$, where $E^* = (E_1\ E_3\ E_2\ \ldots\ E_{m-1}E_{m-2}E_m)^t$ when $m \equiv 0 \ mod \ 2$, (resp., $E^* = (E_1\ E_3\ E_2\ \ldots\ E_mE_{m-1})^t$ when $m \equiv 1 \ mod \ 2$). Since, for $2 \leq f \leq m-1$, the difference between $wt(E_f)$ and $wt(E_{f+1})$ is at most 4 and the difference between $wt(r_f)$ and $wt(r_{f+1})$ of the subarray $L_2T_5T_6$ is at least 5, the weights of the vertices (rows) in the subarray $E^*L_2T_5T_6$ are pairwise distinct. □

Note that when n is odd, the construction of Case 1 (as given in the proof of Theorem 3.2) provides another graph that is antimagic, but slightly different to the one obtained in Case 2 above (it is not an antimagic generalized toroidal antiprism graph).

The generalized toroidal antiprism graph $T_{C_4}^4$ with antimagic labeling is illustrated in Figure 3.

1	2	9	10	13	14
3	4	9	11	15	16
5	6	10	12	17	18
7	8	11	12	19	20
1	3	21	22	25	26
2	5	21	23	27	28
4	7	22	24	29	30
6	8	23	24	31	32
13	15	33	34	37	38
14	17	33	35	39	40
16	19	34	36	41	42
18	20	35	36	43	44
25	27	37	39	45	46
26	29	38	41	45	47
28	31	40	43	46	48
30	32	42	44	47	48

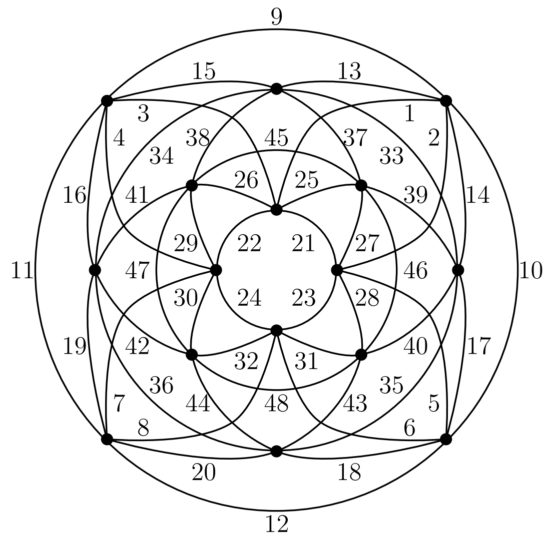

Figure 3. The generalized toroidal antiprism graph $T^4_{C_4}$ with antimagic labeling.

We conclude with a corollary that follows immediately from the corresponding theorems and corollaries when $G = K_m$ is replaced by K_2. Note that the constructions of Case 1 in the proof of Theorem 3.2 works for $T^n_{K_2}$, for any $n \geq 3$. The details are omitted here.

Corollary 3.5. (i) *The graph $A^n_{K_2}$, $n \geq 2$, is antimagic.*
(ii) *The graph $B^n_{K_2}$, $n \geq 2$, is antimagic.*
(iii) *The graph $TW^n_{K_2}$, $n \geq 2$, is antimagic.*
(iv) *The graph $T^n_{K_2}$, $n \geq 3$, is antimagic.*

4. Conclusion

We conclude with a challenge to prove or disprove the following open problem.

Open Problem 1. *Is it possible to construct antimagic labelings for all A^n_G, $n \geq 2$, and T^n_G, $n \geq 3$, where G is any regular graph?*

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