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Independent strong domination in complementary prisms

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Abstract

Let $G = (V, E)$ be a graph and $u, v \in V$. Then, u strongly dominates v if (i) $uv \in E$ and (ii) $deg(u) \geq deg(v)$. A set $D \subset V$ is a strong-dominating set of G if every vertex in $V - D$ is strongly dominated by at least one vertex in D. A set $D \subseteq V$ is an independent set if no two vertices of D are adjacent. The independent strong domination number $i_s(G)$ of a graph G is the minimum cardinality of a strong dominating set which is independent. Let $\bar{G}$ be the complement of a graph G. The complementary prism $G\bar{G}$ of G is the graph formed from the disjoint union of G and $\bar{G}$ by adding the edges of a perfect matching between the corresponding vertices of G and $\bar{G}$. In this paper, we consider the independent strong domination in complementary prisms, characterize the complementary prisms with small independent strong domination numbers, and investigate the relationship between independent strong domination number and the distance-based parameters.

1. Introduction

Graph theoretic techniques provide a convenient tool for the investigation of networks. It is well-known that an interconnection network can be modeled by a graph with vertices representing sites of the network and edges representing links between sites of the network. Therefore various network problems can be studied by graph theoretical methods. The study of domination in

graphs is an important research area, perhaps also the fastest-growing area within graph theory. The reason for the steady and rapid growth of this area may be the diversity of its applications to both theoretical and real-world problems. For instance, dominating sets in graphs are natural models for facility location problems in operations research. Research on domination in graphs has not only important theoretical signification, but also varied application in such fields as computer science, communication networks, ad hoc networks, biological and social networks, distributed computing, coding theory, and web graphs. Domination and its variations have been extensively studied [1, 6, 7, 9, 12, 14, 16, 17]. In general, the concept of dominating sets in graph theory finds wide applications in different types of communication networks. A broadcast from a communication vertex is received by all its neighbors. This is captured by the notion of domination in a graph. The minimum dominating set of sites plays an important role in the network for it dominates the whole network with the minimum cost. A thorough study of domination appears in [16, 17]. Independent sets play an important role in graph theory and other areas like discrete and combinatorial optimization. They appear in matching theory, coloring of graphs, and in trees. In a communication graph, an independent set consists of vertices that cannot communicate with one another directly. From an application point of view, independent and dominating set in a communication network are important structures, and many optimization approaches rely on these. In some sense, one could say that the domination and independence based parameters reveal an underlying efficient and stable communication network. Among the domination-type parameters that have been studied, the independent strong domination number is one of the fundamental ones. A set $D \subseteq V$ is a dominating set if every vertex not in D is adjacent to at least one vertex in D. A set $D \subseteq V$ is a strong dominating set if every vertex u not in D is adjacent to a vertex v in D where $deg(v) \geq deg(u)$. The domination number of G (strong domination number), denoted $\gamma(G)(\gamma_s(G))$ is the minimum size of a dominating set (strong dominating set) of G. A set $D \subseteq V$ is an independent set if no two vertices of D are adjacent. The independent domination number of G (independent strong domination number), denoted $i(G)(i_s(G))$ is the minimum size of an independent dominating set (independent strong dominating set) of G. Then a minimal independent strong dominating set of cardinality $i_s(G)$ is called an $i_s(G) - set$ [2, 5, 11, 13]. It follows from the definitions that for any graph G, $\gamma(G) \leq i(G) \leq i_s(G)$ and $\gamma(G) \leq \gamma_s(G) \leq i_s(G)$[4]. Every graph admits an independent strong dominating set [11]. For example, to find an independent strong dominating set, say D, in a graph G, the following algorithm can be applied:

$S := \emptyset$
$D := \emptyset$
while S $\neq$ V
begin
    Let $v \in \{v \in V - S|$ deg(v) is as big as possible$\}$;
    $S := S \cup N[v]$;
    $D := D \cup \{v\}$
end;

The concept of strong domination was introduced by Sampathkumar and Pushpa Latha in [5] by the following motivation. Consider a network of roads connecting a number of locations. In such a network, the degree of a vertex v is the number of roads meeting at v. Suppose $deg(u) \geq deg(v)$.

Naturally, the traffic at u is heavier than that at v. If we consider the traffic between u and v, preference should be given to the vehicles going from u to v. Thus, in some sense, u strongly dominates v. In [11], it is shown that the problems of computing i_s is NP-hard, even for bipartite graphs. Since computing the independent strong domination of a graph is NP-hard in general, it becomes an interesting question to calculate the independent strong domination for some special classes of interesting or practically useful graphs. In the following two sections we will deal with this question.

In this paper, we consider finite undirected graphs without loops and multiple edges. The order of G is the number of vertices in G. The open neighborhood of v is $N(v) = \{u \in V | uv \in E\}$ and the closed neighborhood of v is $N[v] = \{v\} \cup N(v)$. For a set $S \subseteq V$, $N(S) = \bigcup_{v \in S} N(v)$ and $N[S] = N(S) \cup S$. The degree of a vertex v is $deg_G(v) = |N(v)|$. A vertex of degree zero is an isolated vertex or an isolate. A leaf or an endvertex or a pendant vertex is a vertex of degree one and its neighbor is called a support vertex. The maximum degree of G is $\Delta(G) = max\{deg_G(v) | v \in V\}$. Define V_Δ as $\{v \in V | deg(v) = \Delta(G)\}$. For $S \subseteq V$, the subgraph of G induced by S is denoted by $G[S]$. The distance $d(u, v)$ between two vertices u and v in is the length of a shortest path between them. If u and v are not connected, then $d(u, v) = \infty$, and for $u = v$, $d(u, v) = 0$. The eccentricity of a vertex v in G is the distance from v to a vertex farthest away from v in G. The diameter of G, denoted by $diam(G)$, is the largest distance between two vertices in V [8, 10].

Complementary prisms are a deeply intriguing family of graphs and the study in complementary prisms is just beginning. Complementary prisms were first introduced by Haynes, Henning, Slater and Van der Merwe in [15]. For a graph G, its complementary prism, denoted by $G\bar{G}$, is formed from a copy of G and a copy of $\bar{G}$ by adding a perfect matching between corresponding vertices. For each $v \in V(G)$, let $\bar{v}$ denote the vertex v in the copy of $\bar{G}$. Formally $G\bar{G}$ is formed from $G \cup \bar{G}$ by adding the edge $v\bar{v}$ for every $v \in V(G)$. For example, if G is a 5-cycle, then $G\bar{G}$ is the Petersen graph. Complementary prisms form an interesting family of graphs, generalizing the concept of graph products as well as graphs such as the Petersen graph. Hence parameters for these graphs are important to study. Independent and domination based parameters of complementary prims were also studied in [18, 19].

Let $G\bar{G}$ be the complementary prism of a graph $G = (V, E)$. For notational convenience, we let $\bar{V} = V(\bar{G})$. Note that $V(G\bar{G}) = V \cup \bar{V}$, and to simplify our discussion of complementary prisms, we say simply G and $\bar{G}$ to refer to the subgraph copies of G and $\bar{G}$, respectively, in $G\bar{G}$. Also, for a vertex v of G, we let $\bar{v}$ be the corresponding vertex in $\bar{G}$, and for a set $X \subseteq V$, we let $\bar{X}$ be the corresponding set of vertices in $\bar{V}$.

The paper proceeds as follows. In the following section, existing literature on independent strong domination is reviewed. The independent strong domination numbers for specific types of graphs and complementary prism $G\bar{G}$ when G is a specified family of graphs are computed and exact formulae are derived. Independent strong domination numbers for the graphs and vertices with specific distance-based parameters are investigated. The graphs and vertices for which i_s is small are characterized.

2. Independent strong domination

2.1. Basic results

Theorem 2.1 ([11]). *For any graph G, $i_s(G) \leq n - \Delta(G)$.*

Theorem 2.2 ([11]). *Let G be a graph. Then $i_s(G) = n - \Delta(G)$ if and only if $V - N(v)$ is independent for every vertex $v \in V_\Delta$.*

Theorem 2.3. *For a graph G of order n, if G has diameter one, then $i_s(G) = 1$.*

Proof. If G has diameter one, then G is the complete graph on n vertices in which all vertices are adjacent to each other and all vertices having the same vertex degree $n - 1$. Therefore, the proof is immediate. □

Theorem 2.4. *Let G be a graph of order n. Then, $i_s(G) = 1$ if and only if G has a vertex with eccentricity one.*

Proof. The sufficiency is immediate since if G has a vertex v with eccentricity one, then this implies that vertex v is adjacent to all other vertices of G with $deg_G(v) = n - 1$. Thus, the set including only the vertex v is the $i_s(G)$-set and strongly dominates V. Now, suppose that $i_s(G) = 1$. This implies that G has a support vertex v that dominates V. Since $N_G[v] = V$, vertex v has eccentricity one. This establishes the necessity. □

2.2. Specific families

We begin our investigation of independent strong domination for this subsection by computing its value for several well-known classes of graphs. We use $\lceil x \rceil$ to denote the least integer not less than x.

Observation 2.1. *The independent strong domination of*

(a) *the complete graph K_n is 1;*
(b) *the null graph $\bar{K}_n$ is n;*
(c) *the star $K_{1,n}$ is 1;*
(d) *the path P_n is $\lceil n/3 \rceil$;*
(e) *the cycle C_n is $\lceil n/3 \rceil$;*
(f) *the wheel W_n is 1;*
(g) *any complete multipartite graph of order p and least partite set of order r is r;*
(h) *the comet $C_{s,t}$ is $1 + \lceil (t-2)/3 \rceil$, where s and t are positive integers, the comet $C_{s,t}$ denotes the tree obtained by identifying the center of the star $K_{1,s}$ with an end-vertex of P_t, the path of order t. So $C_{s,t} \cong K_{1,s}$ and $C_{1,p-1} \cong P_p$.*

2.3. Complementary prisms

We next determine the independent strong domination number of the complementary prism $G\bar{G}$ when G is a specified family of graphs. We shall make use of the following lemmata.

Lemma 2.1. (a) $\gamma(P_n) = \lceil n/3 \rceil$;
(b) $\gamma(C_n) = \lceil n/3 \rceil$;
(c) $\gamma(\bar{C}_n) = 2$ *for $n > 3$;*

Definition 2.1. *Let G and H be two graphs of order n and m, respectively. The corona product $G \circ H$ is defined as the graph obtained from G and H by taking one copy of G and n copies of H, and then joining the ith vertex of G to every vertex in the ith copy of H.*

The following lemma gives a formula for the independent strong domination number of corona of two graphs in terms of the independent strong domination numbers of these two graphs.

Lemma 2.2. *Let G and H be two graphs of order n and m, respectively. Then, $i_s(G \circ H) = i_s(G) + (n - i_s(G))i_s(H)$.*

Proof. In a graph $G \circ H$, for a vertex $v \in V(G)$, $deg_{G \circ H}(v) \geq m$; and for a vertex $u \in V(H)$, $deg_{G \circ H}(u) \leq m$. Therefore, it is easy to check that $deg_{G \circ H}(v) \geq deg_{G \circ H}(u)$, where $v \in V(G)$, $u \in V(H)$. If we take the vertices of the $i_s(G)$-set to the $i_s(G \circ H)$-set, then there remain $n - i_s(G)$ copies of H of which the vertices are non-dominated. Since, all vertices of G is dominated, it is necessary to include $(n - i_s(G))i_s(H)$ vertices in $i_s(G \circ H)$-set yielding $i_s(G \circ H) = i_s(G) + (n - i_s(G))i_s(H)$, and taking any subset of $V(G \circ H)$ with cardinality less than $i_s(G) + (n - i_s(G))i_s(H)$ does not yield an $i_s(G \circ H)$-set. $\square$

Theorem 2.5. (a) *If $G = K_n$, then $i_s(G\bar{G}) = n$.*
(b) *If $G = tK_2 (t > 1)$, then $i_s(G\bar{G}) = t + 1$.*
(c) *If $G = K_t \circ K_1$, then $i_s(G\bar{G}) = t + 1$.*
(d) *If $G = K_{1,n}$, then $i_s(G\bar{G}) = 2$.*
(e) *If $G = K_{m,n}$ where $2 \leq m \leq n$, then $i_s(G\bar{G}) = m + 1$.*
(f) *If $G = C_n (n > 3)$, then $i_s(G\bar{G}) = \lceil (n + 4)/3 \rceil$.*
(g) *If $G = W_n (n > 3)$, then $i_s(G\bar{G}) = 3$.*
(h) *If $G = P_n (n > 3)$, then $i_s(G\bar{G}) = \lceil (n + 4)/3 \rceil$.*

Proof. To prove (a), for $G = K_n$, the complementary prism $G\bar{G}$ is the corona $K_n \circ K_1$. Thus, by Lemma 2.2 and Observation 2.1(a), the proof is straightforward.

To prove (b), label the $2t$ vertices of $V(G)$ as a_i, b_i where $1 \leq i \leq t$ such that $a_i b_i \in E(G)$. Let $A = \{a_1, ..., a_t\}$ and $B = \{b_1, ..., b_t\}$. Since $deg_{G\bar{G}}(\bar{a}_i) = deg_{G\bar{G}}(\bar{b}_i) = 2t - 1 > deg_{G\bar{G}}(a_i) = deg_{G\bar{G}}(b_i)$ for $\forall i \in [1, .., t]$ and $t > 1$, both vertices $\bar{a}_k$ and $\bar{b}_k$ are included in $i_s(G\bar{G})$-set to strongly dominate $V(\bar{G})$ and the vertices a_k and b_k of G. Then, any combination of the nondominated vertices with cardinality $t - 1$ in which including the vertices a_r and b_s where $r \neq s, k, s \neq r, k$ ($\forall r, s \in [1, .., t]$) can be in $i_s(G\bar{G})$-set. Consequently, we have $i_s(G\bar{G}) = t + 1$, and taking any subset of $V(G\bar{G})$ with cardinality less than $t + 1$ does not yield an $i_s(G\bar{G})$-set.

To prove (c), let $G = K_t \circ K_1$. If $t = 1$, then $G = K_2$ and from (a), $i_s(G\bar{G}) = 2$. Assume that $t \geq 2$, and label the vertices of G as follows: let $A = \{a_i | 1 \leq i \leq t\}$ be the set of t vertices that induce the subgraph K_t of G, and let $B = \{b_i | 1 \leq i \leq t\}$ be the end-vertices in G adjacent to the vertices in A such that $a_i b_i \in E(G)$. Let I be an $i_s(G\bar{G})$-set. Since $deg_{G\bar{G}}(\bar{b}_k) = \Delta(G\bar{G})$ for a vertex $\bar{b}_k$ $(k \in [1, .., t])$ in $G\bar{G}$, $\bar{b}_k \in I$ to strongly dominate the vertices of $\bar{B}$, $\bar{A} \setminus \{a_i\}$, and the vertex $b_i \in B$. Among the remaining non-dominated vertices, since I should be independent, being $deg_{G\bar{G}}(\bar{a}_i) = t < deg_{G\bar{G}}(a_i) = t + 1 (\forall i)$, $a_k \in I$ to strongly dominate $\bar{a}_k \in \bar{A}$ and $A \setminus \{a_k\}$. To strongly dominate the nondominated vertex set $B \setminus \{b_k\}$, all $t - 1$ vertices of $B \setminus \{b_k\}$ should be included in the independent set I. Then, we have $I = \bar{b}_k \cup a_k \cup B \setminus \{b_k\}$ with cardinality $|I| = t + 1$, and taking any other subset of $V(G\bar{G})$ with cardinality less than $t + 1$ does not yield an $i_s(G\bar{G})$-set.

To prove (d), since G is a star, the support vertex t in G is an isolated vertex $\bar{t}$ in $\bar{G}$, and a leaf in $G\bar{G}$. Denote the n leaves of G by $\{u_i | 1 \leq i \leq n\}$. The leaves in G form a complete graph on n vertices in $\bar{G}$. The support vertex t of G has a vertex degree $deg_{G\bar{G}}(t) = n + 1$ in $G\bar{G}$ which is greater than or equal to of all its neighboring vertices, $deg_{G\bar{G}}(t) > deg_{G\bar{G}}(\bar{t}) = 1$ and $deg_{G\bar{G}}(t) = deg_{G\bar{G}}(u_i)$ if $n = 1$; $deg_{G\bar{G}}(t) > deg_{G\bar{G}}(u_i)$ if $n > 1$. Let I be an $i_s(G\bar{G})$-set. Then, $t \in I$ to strongly dominate the set $N_{G\bar{G}}[t]$. Include one of the vertices $\bar{u}_i$ for some i in I to strongly dominate the remaining non-dominated vertices that all are of the same vertex degree n. Hence, $I = \{t, u_i\}$ with cardinality $|I| = 2$, and taking any other subset of $V(G\bar{G})$ with cardinality less than 2 does not yield an $i_s(G\bar{G})$-set.

To prove (e), let $G = K_{m,n}$ $(2 \leq m \leq n)$, where R and S are the partite sets of G with cardinality m and n, respectively. Let $R = \{r_1, r_2, ..., r_m\}$ and $S = \{s_1, s_2, ..., s_n\}$. The vertices of R and S form complete graphs K_m and K_n on m and n vertices, respectively, in $\bar{G}$.
If $m \leq n$, then this yields $deg_{G\bar{G}}(r) \leq deg_{G\bar{G}}(v)$, where $v = \bar{r}$ or $v = s$ or $v = \bar{s}$. Therefore, if I is an $i_s(G\bar{G})$-set, then one of the vertices of the partite set R is in I. This vertex strongly dominates the partite set S and a vertex $\bar{r}$ of $\bar{R}$. For the non-dominated vertices of $\bar{S}$, I contains one of the vertices of $\bar{S}$, say $\bar{s}$, to strongly dominate K_n. To strongly dominate the yet non-dominated vertices of R and $\bar{R}$, since $deg_{G\bar{G}}(r) > deg_{G\bar{G}}(\bar{r})$ for all $r \in R$ and $\bar{r} \in \bar{R}$, I includes the set R. Henceforth, $I = R \cup \{\bar{s}\}$ with cardinality $|I| = m + 1$, and taking any other subset of $V(G\bar{G})$ with cardinality less than $m + 1$ does not yield an $i_s(G\bar{G})$-set.

To prove (f), if I is an $i_s(G\bar{G})$-set, then I includes a vertex $\bar{u}$ of $\bar{G}$ since $deg_{G\bar{G}}(\bar{u}) = \Delta(G\bar{G})$ for $n \geq 5$. Let $S = V(G\bar{G}) \setminus N_{G\bar{G}}[\bar{u}]$. Since $G\bar{G}[S] = C_{n+1}$, a minimum dominating set of C_{n+1} including a vertex $\bar{v}$ $(\bar{v} \neq \bar{u})$ of $\bar{G}$ belongs to the set I yielding the cardinality of I, $|I| = \gamma(C_{n+1}) + 1 = \lceil (n+1)/3 \rceil + 1$, and taking any other subset of $V(G\bar{G})$ with cardinality less than $\lceil (n+1)/3 \rceil + 1$ does not yield an $i_s(G\bar{G})$-set. For $n = 4$, if I is an $i_s(G\bar{G})$-set, then I includes a vertex u of G since $deg_{G\bar{G}}(u) = \Delta(G\bar{G})$. Let $S = V(G\bar{G}) \setminus N_{G\bar{G}}[u]$. Since $G\bar{G}[S] = \bigcup_{i=1}^{2} P_2$. By Observation 2.1(d) $i_s(P_n) = \lceil n/3 \rceil$ and we have $i_s(G\bar{G}[S]) = 2\lceil 2/3 \rceil = 2$ yielding $|I| = 3$, and taking any other subset of $V(G\bar{G})$ with cardinality less than $|I| = 3$ does not yield an $i_s(G\bar{G})$-set.

To prove (g), let G be a wheel of order $n + 1$ and consider $G\bar{G}$. If I is an $i_s(G\bar{G})$-set, then first, the center vertex $c \in V(G)$ which is adjacent to every other vertex of G and which has the maximum vertex degree in $G\bar{G}$ belongs to I in order to strongly dominate $N_{G\bar{G}}[c] = \{\bar{c}\} \cup V(G)$. Let $S = V(G\bar{G}) \setminus N_{G\bar{G}}[c]$. Eventually, $G\bar{G}[S] = \bar{C}_n$ which is regular. Therefore, a minimum dominating set of $\bar{C}_n$ belongs to I. By Lemma 2.1, we have that $i_s(G\bar{G}) = |I| = 1 + \gamma(\bar{C}_n) = 3$,

and taking any other subset of $V(G\bar{G})$ with cardinality less than $|I| = 3$ does not yield an $i_s(G\bar{G})$-set.

To prove (h), let I be an $i_s(G\bar{G})$-set. For an endvertex $v \in V(P_n)$, since $deg_{G\bar{G}}(\bar{v}) = \Delta(G\bar{G})$, $\bar{v} \in I$. If $S_1 = V(G\bar{G}) \setminus N_{G\bar{G}}[\bar{v}]$, then $G\bar{G}[S_1] = P_n$ with the endvertices of vertex degree $n-2$, 2, and the internal vertices of the same degree 3 in $G\bar{G}$. Therefore, I includes the endvertex $\bar{u}$ with degree $n-2$ strongly dominating an internal vertex for $n > 4$. If $S_2 = V(G\bar{G}) \setminus \{N_{G\bar{G}}[\bar{v}] \cup N_{G\bar{G}}[\bar{u}]\}$, then $S = P_{n-2}$. Then, clearly $I = \{\bar{v}\} \cup \{\bar{u}\} \cup D$, where D is a minimum dominating set of P_{n-2}. Thus, we achieve $i_s(G\bar{G}) = |I| = 2 + \gamma(P_{n-2}) = 2 + \lceil (n-2)/3 \rceil$. For $n = 4$, a minimum dominating set of $G\bar{G}[S_1] = P_n$ with cardinality $\gamma(P_n) = \lceil n/3 \rceil = 2$ including an internal vertex of degree 3 and an endvertex of degree 2. Thus, we have $i_s(G\bar{G}) = 1 + \lceil n/3 \rceil = 3$, and taking any other subset of $V(G\bar{G})$ with cardinality less than $\lceil (n+4)/3 \rceil$ does not yield an $i_s(G\bar{G})$-set. $\square$

2.3.1. Small values

Theorem 2.6. *For a graph G of order n and its complementary prism $G\bar{G}$, $i_s(G\bar{G}) = 1$ if and only if $n = 1$.*

Proof. The sufficiency is immediate since if $n = 1$, then $G\bar{G} = K_2$. Thus, by Obsevation 2.1(a), $i_s(G\bar{G}) = 1$. Now, suppose that $i_s(G\bar{G}) = 1$. Then, any $i_s(G\bar{G})$-set is in either $V(G)$ or $V(\bar{G})$ implying that G has only one vertex. Hence $G = K_1$. This establishes the necessity. $\square$

Theorem 2.7. *For a graph G of order n, if either G or $\bar{G}$ has diameter one, then $i_s(G\bar{G}) = n$.*

Proof. If either G or $\bar{G}$ has diameter one, then $G\bar{G}$ is the corona $K_n \circ K_1$. Therefore, by Lemma 2.2 and Observation 2.1(a), the proof is immediate. $\square$

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